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An Uncertainty analysis of a color tolerance database

Mitchell Balonon-Rosen

AN UNCERTAINTY ANALYSIS OF A
COLOR TOLERANCE DATABASE

by

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A thesis submitted in partial fulfillment of the requirement for the degree of Master of Science in the Center for Imaging Science in the College of Imaging Arts and Sciences of the Rochester Institute of Technology

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The M.S. Degree Thesis of Mitchell Balonon-Rosen has been examined and approved by the thesis committee as satisfactory for the thesis requirement for the Master of Science degree

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Submitted to the Center for Imaging Science in partial fulfillment of the requirements for the Master of Science Degree at the Rochester Institute of Technology

ABSTRACT

A database designed to develop, evaluate and compare color-difference formulae, particularly for use within the suprathreshold industrial-sized range, so named for its importance in many commercial transactions, has been under development at the Munsell Color Science Laboratory. The database consists of two independent data sets, previously published. The sets and the experimental conditions under which they were derived were analyzed to determine the appropriateness of pooling the data. The earlier of the two studies had associated statistics which indicated a population of highly precise observers. The latter study's population displayed a significantly greater level of uncertainty. The current work concluded that the increase in observer imprecision was a consequence of increased task difficulty. A median filtering of raw individual observer responses improved interpretability of the data and enabled pooling of the sets. The pooled results were compared with various widely used color-difference formulae of which CMC(1:1) had the highest correlation.

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It is with great appreciation that I acknowledge a few of the people who helped me and stood by me through the various stages of this work. First, I would like to single out my thesis advisor, Dr. Roy Berns, with whom it has been a source of pride to be associated. Dr. Bern's stewardship of the Munsell Color Science Laboratory provides a comfortable setting for serious and important scholarship. Since my departure from RIT, Dr. Berns has provided me with a level of long distance help which speaks volumes about his dedication to his students' successes. The completion of this thesis is indeed his victory as it is mine.

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Dedication

December 6, 1992

This thesis is dedicated to my family: to my parents, Leonard and Adelaide Rosen, who have always believed one hundred percent in their children, to Alma, my wife and friend, who has supported me in this effort for years, to our delightful daughter Marissa, who's entrance on the scene inspired me to finish this, and to our new son Peter, born 14 days ago, may he never know a world where Daddy is late coming home because he is working on his master's thesis.

Table of Contents

List of Tables

List of Figures

I. Introduction

A multi-phase research project has been underway at the Munsell Color Science Laboratory to create a database of experimentally derived human color difference responses for a large subject population with respect to a wide sampling of color-space. Two independent studies, Phase I[1,2] and Phase II[3], examined a total of nine CIELAB[7] color directions in the vicinity of 19 color centers. Although the studies shared similar experimental designs, they produced very different confidence statistics. The purpose of the current work was to evaluate differences between the two studies and determine if pooling the data was appropriate.

Phases I and II took advantage of an experimental method which enabled a quantitative comparison of color-differences throughout color space. A color-difference standard called the anchor pair was used for these comparisons. It consisted of two near-neutral painted aluminum samples, differing in all three CIELAB dimensions, with a color-difference magnitude of approximately 1 ΔE^*_{ab} unit, and mounted on a gray background. Color-differences visually matching the anchor pair were dubbed *industrial-sized* because of their importance for many commercial transactions. The anchor pair thus measured one industrial-sized color-difference unit. Test-pairs were of similar construction to the anchor pair. Observers were asked to make binary forced choice determinations as to whether the perceived color-differences of test-pairs were greater than or less than that associated with the anchor pair.

The inconsistency between currently available color-difference scales such as CIELAB and human perceived magnitudes, particularly in the realm of industrial-sized color-differences, was the main motivation behind these earlier investigations.[37-39] Snyder,[1] Alman et al.,[2] Reniff,[3] and Berns et al.[4] have produced a body of work describing the background, implementation and results of Phases I and II. They have justified the need for these studies and have thoroughly explained the methodology used for the experimental design and the data analysis. In order to put the current work in proper context, the aforementioned papers should be studied.

The statistical analysis method for Phase I and Phase II was Probit analysis.[5] Probit was designed for determining population tolerance levels for quantal experiments where observers responded normally with respect to a stimulus level and where individual observations were completely independent. The stimulus to which observers reacted in Phases I and II was test-pair color-difference. The tolerance level sought by the experiments was the level of CIELAB color-difference corresponding to one industrial-sized color-difference unit at various points in color-space and in particular color-space orientations. The term *T50* was used in these studies to describe the median tolerance level as determined by Probit analysis, *color center* signified locations in color-space about which data were taken, and the term *color vector* was used to designate the tri-valued entity comprising the resultant T50 magnitude, its associated color center and its color-space orientation.

Phase I statistical analysis showed an observer population responding normally for 82% of the vectors.[1] Unlike these encouraging statistics, Phase II results were cause for concern. Only 47% of the T50's were associated with high confidence measurements.[3]

The current project was mandated the responsibility to identify the differences between Phases I and II and to determine if and how the data could be pooled. Several experiments were designed to test theories about the change in confidence statistics. These experiments helped to narrow the list of probable primary contributors. The most likely causes for the decrease in statistical confidence were identified as follows: color-space orientation of color-difference test-pairs and the color distance of test-pair colors from the anchor pair. It was concluded that these factors resulted in making Phase II a more difficult task for observers.

A median filtering technique was developed for use on the raw observer responses. The effect of the median filter was to reduce within-observer noise so that the Probit analysis could properly measure between-observer variation. *A priori* knowledge of how individuals react to locally-increasing color-differences was used as rationale for applying the filter.[6] Phase I T50 metrics were changed little between the filtered and unfiltered responses where maximum magnitude difference was 0.03† ΔE^*_{ab} units. 91% of the Phase II filtered T50's were within 0.10 ΔE^*_{ab} units of the unfiltered values. Filtered Phase I data showed 34† of its 45 color vectors passing

† The unfiltered Phase I and Phase II statistics being compared with the filtered statistics are not the same as those reported by Snyder[1], Alman et al.[2] and Reniff[3], nor are the filtered statistics the same as those reported by Berns et al[4]. This is because the raw data was relogged for the current study. For further explanation, see the sections **Phase I, Phase II** and **This Thesis,** in the **Background** chapter.

confidence tests, a slight decrease from 35† passing prior to the filtering. Filtered Phase II data showed an increase to 86† from 56† of its 119 unfiltered vectors.

The filtered results were compared to the following list of color-difference formulae: XYZ[8] Euclidean distance, CIELAB[7], CIELUV[7], SVF[9], FMC2[10,11], BFD(1:1)[12,13], CMC(1:1)[14,15], and the NBS Unit of Color-Difference[16,17]. CMC(1:1) was found to have the closest association with industrial-sized color differences.

II. Background

A. Uniform Color-Spaces and Color-Difference Formulae

In 1931 the CIE established the standard observer and the ability to calculate trichromatic responses. This provided the world with unambiguous color specification. A color sample described by XYZ tristimulus values should visually match another sample described by the same XYZ tristimulus values under identical viewing conditions. The ability to measure colors by means of a spectrophotometer and to transform measurements to XYZ values created a "universal and fundamental language of color."[23] The 1931 standard observer was greatly important for the growth of color science. As the Handbook of Colorimetry[23] pointed out, "Students of history agree that man's progress was slow until he had developed a language that enabled him to impart to others the experience that he had just acquired."

By 1934, transformations of the XYZ system were being developed for superior correlation between calculated distances and human visual perception.[17] Known as uniform color spaces or uniform color scales, many XYZ transformations have been offered over the past sixty years. Earliest attempts at improving the non-uniform nature of XYZ space concentrated on the two-dimensional projection known as the chromaticity diagram. MacAdam, one of the original researchers for the color science "Holy Grail" of a universal uniform color space, reminisced:[25]

> Analogous to Mercator charts and other kinds of maps of the world that misrepresent the ratios of distances, the chromaticity diagram does not

represent perceptually equal color differences by equal distances between points that represent equally luminous colors. The noticeability of color differences was not considered - very few data were available - when the chromaticity diagram was devised and adopted. However, as soon as it came into use, anomalies were encountered in interpreting the configurations of points on the diagram. Inconsistencies between distances and perceived magnitudes of color differences were evident. The analogy with geographical maps was quickly noted and suggestions were made to change the representation so that equal distances would represent equally noticeable color differences. The hoped-for chromaticity diagram with such properties came to be called "uniform". The search for it has extended over 50 years and seems no nearer its goal than at the beginning. Much of the accumulated evidence indicates that the goal is unattainable that a flat diagram cannot represent equal color differences by equal distances any more than a flat map of the world can represent equal geographical distances by equal distances on the map.

Without a uniform color space it was necessary to perform special investigations for each color about which a color tolerance was to be specified. MacAdam[25] and Billmeyer[26] have described the use of "limit standards" which are chosen as representatives of acceptable "extreme variations."

Many uniform color spaces have been offered. Hunter[17] has given an extensive history to the development of many of these scales. In general, the scales break down into three categories: those deriving from the work of Albert Munsell:[28-31] those in the family line of Judd's 1939 commercial-acceptance measurements;[32] and, those which can be traced back to MacAdam's reports in the early

1940's on just-noticeable differences[33,34] (jnd's). In 1976 the CIE recommended that the color community use either of two color-difference formulae,[7,36] CIELAB or CIELUV. CIELAB is a member of the Munsell family, CIELUV derives from MacAdam just-noticeable difference data. As these and their derivations have become the most dominant uniform color spaces, the lack of best fit for industrial-sized color-differences by either has proven troublesome.[37-39]

B. Overview of Color-Difference Equations

In 1969 Wright wrote "the preference for one [color-difference] formula over another is likely to be determined by its practical convenience and ease of application rather than because of some superior visual validity."[40] Over the years and between industries, these criteria have had inconsistent interpretation. For example, equations once thought too complicated for human calculation or for analog circuitry have become less intimidating as computers and digital circuitry have become commonplace. Yet, simplicity has continued to be a driving force. Visual factors important to a particular niche have been incorporated into formulae only to find indifference from the color-difference marketplace. Historical precedence, as well, has always had a marked influence on the use of metrics. "Practical convenience" is often defined by the common language, regardless of its appropriateness to the problem at hand.

The color-difference formulae compared in this thesis were chosen because they are in wide use today. One exception is the NBS unit of color-difference, chosen because it once enjoyed wide use and was derived for industrial-sized color-differences. Appendix B lists the formulae for these equations.

The NBS unit of color-difference[16,17] also known as the Judd color-difference unit, is associated with the Judd-Hunter or Modified Judd formula, derived by Hunter in 1942. According to Hunter, "differences of less than one unit are usually not important in commercial transactions."[17] This unit was based on Judd's 1939 formula,[32] a summary of dye house color-matching investigations. Hunter transformed the 1939 formula to his "alpha-beta" rectangular chromaticity diagram and used an additional 10,000 observations of tile samples. The formula included a "gloss factor," considered by Hunter as late as 1987 to be unique among uniform color spaces.[17]

The FMC-2 formula[11] was based upon the FMC-1 metric.[41] The earlier formula was a three-dimensional fit to the results of the MacAdam series of jnd studies. FMC-2 added two factors to better conform with the Simon-Goodwin type of lightness and chromaticness differences.[42,43] The first factor was specifically developed for textile industry use. It simulated the "swelling/ shrinking behavior of Simon-Goodwin chromaticness differences."[11] The other factor was developed to constrain grays to conform to Simon-Goodwin lightness differences.

By 1976 it was recognized that as many as 20 different color-difference formulae were being used world wide.[7] While many studies were made comparing the various available formulae, no clear winner was emerging. At the time, particularly in Europe, there was a move to standardize on the Adams-Nickerson formula[44,45] also known as the Adams Chromatic Value formula or

ANLAB. A disadvantage to the formula was the set of non-invertible quintic expressions relating fundamental factors to the CIE XYZ tristimulus values. In order to estimate these factors, table lookups and interpolations were necessary. A simplification using cube-root relationships was shown to deviate from the original insignificantly and grew into CIELAB.[7]

CIELUV[7] was derived as a modification to the 1964 CIE U*V*W* formula.[46] Both CIELUV and the 1964 formulas had associated chromaticity diagrams with desirable properties for additive systems. Industries that worked with additive colors, such as color television, found great functionality in chromaticity diagrams which preserved a colinear relationship between the chromaticities of any two colored lights and the chromaticity of their weighted combination. The position of the resultant chromaticity was directly calculable from the relative radiance levels. No such diagram was available for CIELAB. CIELAB and CIELUV shared a common lightness component, L*.

CMC(l:c),[14] disclosed in 1984, was an improvement to the JPC79 formula[47,48] which, in turn, was a modification of ANLAB.[52] JPC79 was based on acceptability results obtained in one study.[12] Under the direction of the Society of Dyers and Colourists' Colour Measurement Committee, for which it was named, CMC(l:c) was formed to remove anomalies introduced in lightness differences between very dark colors and anomalies introduced in hue differences between near-neatral colors. The CMC(l:c) formula also added the 'l' and 'c' attributes which allowed application specific setting of relative tolerances to lightness and chroma differences.

Reported in 1986, the SVF[9] color space was an attempt to "test whether it was possible to improve the quantitative description of color-differences by introducing a few physiological assumptions about signal processing in the eye." In particular, three aspects of contemporary understanding of eye/brain color processing were addressed: the relationship between the amount of light incident upon the three cone pigments and the resultant receptor response; the relative sensitivities of the three cone types and their saturation characteristics; and, the opponency mechanism for chromatic vision. The SVF formula was a modification of the Munsell Renotation System.[49]

BFD(l:c)[12,13] was the outcome of comparing 11 color-difference formulae to a combined database of 15 published perceptibility and acceptability data sets. A total of 132 color centers were used. While CMC(l:c) was shown to perform best, systematic errors were identified. A modification of CMC(l:c) became BFD(l:c). BFD(l:c) was designed to be similar in structure to the CMC(l:c) formula with newly derived coefficients and an additional term incorporated to correct the claimed CMC(l:c) defect of always orienting discrimination ellipsoids toward the achromatic axis of CIELAB.

Of the above formulae, FMC-2 and CMC(l:c) do not always calculate the same color-difference between two colors when the assigned the role of standard is changed. This has often been considered an undesirable characteristic. The use of weighted CIELAB ΔE^*_{ab} components, enjoyed by both CMC(l:c) and BFD(l:c), is an approach that Alman and coworkers found to give superior performance.[2]

C. Introduction to Phases I and II

"It may be noted that all color-difference formulas are designed to provide results that describe or fit well one or another body of visual data (but not more than one, since these data sets are not consistent with one another)."[35] Here Billmeyer and Saltzman make reference to one of the most important motivations for this study and its predecessors. Uniform color spaces were derived from and fit best one or another data set. As described above, most color-difference formulae can be traced back to one of three data sets. It follows that each formula is best suited to deliver psychophysically accurate color-difference magnitudes for differences which are similar to those comprising its associated data set. While the Judd family of color spaces actually did derive from commercially important color-differences, Munsell-based formulae and MacAdam just-noticeable difference color spaces did not. Munsell spacing is very large with respect to industrial-sized color-differences. Just-noticeable differences are very small with respect to industrial-sized color-differences. Recall that the two current CIE recommended color-difference formulae, CIELAB and CIELUV, derive respectively from Munsell and MacAdam jnd spaces. Phases I and II were designed to gather data about human perception of industrial-sized color-differences.

In 1989, Alman et al.[2] reported the results of Snyder's[1] color tolerance experiment. This experiment has been referred to as Phase I in this thesis. A pair of near-neutral paint samples that varied in all three CIELAB dimensions with a color-difference of approximately one ΔE^*_{ab} was prepared. This pair, considered to have a typical industrial-sized color-difference, was chosen as the anchor pair. Using a psychophysical technique of paired comparison, test-pairs were compared to the anchor pair. Fifty color-normal observers volunteered for the experimental task. Observers viewed a randomized set of test-pairs. For each pair, observers were given the forced choice between designating "pass", if the perceived magnitude of the test-pair color-difference were smaller than that of the anchor pair, or "fail", if otherwise. The two painted colors used to create the test-pairs were carefully chosen so that they were orientated in one of five color directions. For Phase I, each vector was associated with one of nine distinct color centers.

The Reniff[3] study, referred to as Phase II, was a follow-up of the earlier Phase I work. Again, fifty color-normal volunteers were assembled. Conceptually, the task was identical to Phase I. Observers were asked to accept a test-pair if its color-difference were smaller than the anchor pair's and reject it if its color-difference were larger. The anchor pair was the same standard as had been used in Phase I. Test-pair physical dimensions were likewise identical to those used in Phase I. Phase II vectors were oriented in one of seven color directions and associated with one of 17 color centers.

D. Probit Analysis

Observer rejection rates were processed through Probit statistical analysis[5] which uses a maximum likelihood model. This procedure was used to estimate the magnitude in ΔE^*_{ab} units of the color-difference at each color center and along each vector direction which would have been perceptually equivalent to the anchor pair's color-difference. Referred to as the T50, this equivalent CIELAB color-difference is an estimate of that which would have been rejected by exactly 50% of the population.

The SAS[54] computer statistical package was used to perform the Probit analysis. In addition to the T50 values the SAS program calculated for each vector an associated σ (standard deviation), a χ^2 value and a χ^2 confidence value. This χ^2 confidence value indicated the probability that the true χ^2 were greater than the reported χ^2. Each T50 value also had an associated fiducial limit range, similar to a confidence interval.

Probit was designed for situations where it would be impractical or impossible to implement a method of limits analysis. An experiment utilizing Probit analysis must meet the following criteria: discrete stimulus levels must be presented to subjects; the subject population should respond in a normal manner to the stimulus; and, each individual response must be completely independent of all others.

The Phase I and Phase II experiments were quantal in nature. Subjects were asked to respond to discrete color-difference magnitudes associated with prefabricated test-pairs. The Probit criterion that the population respond in a normal manner was tested within the analysis for each T50 and the goodness-of-fit was quantified in the χ^2 probability term.

As described above, Probit analysis would have been applied inappropriatly had there been a dependence between responses. The classical text on Probit experiments, written by Finney,[5] did not entertain the possibility that there exist experiments which could reuse subjects to receive multiple stimulus levels. It assumed that this would automatically violate the "independence" criterion. This conservative stance prevented any situation where a residual effect from earlier observations influenced later responses, potentially skewing results. "For the method to be satisfactory, there must be no cumulative effect of doses already given, either as lowering or as increasing the resistance of the subject, a condition which severely limits its applicability."[5] Jameson and Hurvich have noted that perceived color is "systematically dependent on both preceding stimulation and on simultaneous stimulation of the remainder of the visual field."[73] However, the experiment on which Jameson and Hurvich based their claim was primarily concerned with the latter phenomenon followed by postulation that preceding stimulus would have similar effect. Conversely, Berns reported general acceptance in the color science community that it would be unreasonable to "expect hysteresis or build-up for color-difference" observations.[55]

T50 represented that level of stimulus which would have caused positive response in 50% of the population. The stimulus for these studies was color-difference. The T50, in CIELAB ΔE^*_{ab} units, was used to determine the population match to the visual appearance of the anchor pair's color-difference for each color direction at each color center. The χ^2 was representative of the deviation of observer responses from the normality assumption. The χ^2 and the number of degrees of freedom were used to lookup a χ^2 probability term indicating the probability that "another equivalent set taken at random would deviate as much from a normal distribution"[6]. χ^2 probability terms of greater than 5% showed good model fit. The standard deviation was associated with the cumulative normal curve to which the actual responses were fit. Fiducial limits delimited the error range about the T50 for a given level of probability. Fiducial limits were calculated using a 95% confidence level.

The Probit procedure used an iterative process to estimate μ and σ such that

$$f(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \qquad (1)^{62}$$

where x is the stimulus level and f(x) is the population fractional response. For the purposes of these studies, reported T50's were the estimated μ's, reported standard deviations were the estimated σ's, and the χ^2 and χ^2 probability terms indicated how closely the estimated curves fit the raw data. The stimulus, x, was measured in ΔE^*_{ab} units. Equation (1) can be rewritten as follows:

$$f(\Delta E^*_{ab}) = \int_{-\infty}^{\Delta E^*_{ab}} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(\Delta E^*_{ab}-T50)^2/2\sigma^2} \qquad (2)$$

E. Differences Between Color Names in This and Previous Papers

Snyder[1] and Alman et al.,[2] in reporting Phase I, and Reniff,[3] in reporting Phase II, used color names convenient for the purposes of their investigations, but not based on any standard naming conventions. Berns et al.[4] derived the ISCC-NBS color names[22] and issued the following notice:

ISCC-NBS color names are defined for illuminant C and the 1931 standard observer. CIELAB was used as a chromatic-adaption transformation to convert the experimental color-center values based on illuminant D65 and the 1964 supplementary standard observer to the required illuminant and observer. Although CIELAB is well known to be not an accurate adaptation transformation, its use seemed reasonable for the purpose of merely assigning color names.

Table I: Relating currently and previously used color names.

ISCC-NBS color name	Previously used name	Original Phase
Moderate blue	Blue	Phase I
Moderate greenish blue	Cyan	Phase I
Medium gray	Gray	Phase I
Moderate bluish green	Green	Phase I
Light brown	Orange	Phase I
Grayish purple	Purple	Phase I
Dark reddish orange	Red	Phase I
Moderate yellow	Yellow	Phase I
Grayish yellow green	Yellow/Green	Phase I
Black	Black	Phase II
Light bluish green	Blue/Green	Phase II
Moderate reddish brown	Brown	Phase II
Dark bluish green	Green/Blue	Phase II
Brilliant greenish blue	Light Blue	Phase II
Very dark red	Maroon	Phase II
Moderate purplish pink	Pink	Phase II
Dark blue	Violet	Phase II
Light gray	White	Phase II
Strong orange yellow	Yellow/Orange	Phase II

Where ever possible in this paper, the ISCC-NBS names have been used. Table I displays the previously used and current names for the color centers.

F. Phase I

The vector directions used in Phase I appear in Table II. Descriptions of the color centers and the anchor pair, appear in Table III. Five of the nine color centers used corresponded to the CIE recommended centers for coordinated research.[39] Those which fulfill this criteria contain 'yes' in the second column.

Table II: Vector Directions Used in Phase I

Vector Direction Name	CIELAB orientation
A	$-L^*$ to $+L^*$
B	$-a^*$ to $+a^*$
C	$-b^*$ to $+b^*$
D	$-a^*,-b^*$ to $+a^*,+b^*$
E	$-a^*,+b^*$ to $+a^*,-b^*$

Table III: Color Centers Used in Phase I

Color Center	CIE recomd.	L^*	a^*	b^*	ΔE^*_{ab} from Anchor
Anchor		49.21	0.045	5.275	-
Moderate yellow	yes	77.2	2.0	36.0	41.6
Grayish yellow green		64.6	-9.9	13.2	20.0
Moderate bluish green	yes	55.0	-27.7	2.0	28.5
Moderate blue	yes	34.2	-1.0	-28.0	36.5
Grayish purple		45.6	11.4	-12.6	21.5
Moderate greenish blue		49.1	-16.2	-11.5	23.4
Dark redish orange	yes	42.8	34.7	22.8	39.4
Light brown		61.2	13.2	20.0	23.1
Medium Gray	yes	58.2	-0.3	0.8	10.0

A high degree of confidence that the Phase I observer population was consistent and had a distribution equivalent to a cumulative normal response was revealed through the Probit analysis. Only eight of the 45 Phase I vectors revealed statistically unexpected behavior.

Reported statistics in Snyder[1] and Alman et al.[2] were based on Snyder's experimentally derived data. The visual task was for observers to accept or reject test-pairs based on comparison of the color-difference magnitude to that of the anchor pair. When an observer indicated that a test-pair passed, Snyder handwrote a check mark (✓) on a preprinted form next to a number representing the accepted test-pair. When an observer indicated that a test-pair failed, Snyder handwrote an 'ex' mark (X) instead at the same place on the response form. The current research necessitated a return to the original response forms.

Handwritten check marks and ex marks can be extremely hard to distinguish. The use of the two marks to represent opposite responses was a very poor choice.

After examining the results of Snyder's population totals, 317 response frequencies from 50 observers, and comparing them with the current totals, tallied from photocopies of the original 50 forms, it was clear that certain ambiguous marks had been interpreted previously as denoting acceptance or rejection and currently as the opposite. There was no possibility to tell which marks were the ones with which the researchers had disagreed. It was only possible to tell which test-pairs were affected by comparing the total number of rejections tabulated. Table A-I presents the differences between Snyder's totals and the ones used for the current research. Note that Table A-I shows only unfiltered responses. Results of filtered response values based upon the current data were reported by Berns et al.[4] Note also in Table A-I that vector direction B for Moderate

Bluish Green showed a very large difference. This has been determined to be due to a typographical error on Snyder's behalf.

77 of the 317 test-pairs used in Phase I showed a discrepancy between the current and Snyder tallies. 20% of the 45 Phase I T50 values were derived using none of the discrepant test-pairs. The other 36 T50's were derived using at least one of the 77 unagreed upon response frequencies. Ignoring the suspected typographical error demonstrated by Moderate Bluish Green vector B, the largest frequency discrepancy had of magnitude of 3 observer responses and no T50 value changed by more than .03 CIELAB ΔE^*_{ab} units. These differences are considered minuscule and are certainly within experimental error. Only the current data were used for the present research.

G. Phase II

The exceptional Phase I results were used to justify an optimistic, ambitious effort for Phase II utilizing the same experimental approach. Four new vector directions were added to each of the original color centers. Unlike any Phase I vectors, these new directions varied simultaneously in all three dimensions of L^*, a^* and b^*. Also, ten new color centers were investigated. These centers were generally much further in ΔE^*_{ab} distance from the neutral anchor than were the Phase I centers. For these new color centers, seven vector directions were tested. Included were the four new vector directions and three of the original directions. The seven directions investigated by Phase II appear in Table IV.

Table IV: Vector Directions Used in Phase II

Vector Direction Name	CIELAB orientation	Also in Phase I
A	-L* to +L*	yes
B	-a* to +a*	yes
C	-b* to +b*	yes
F	-L*,-a*,-b* to +L*,+a*,+b*	
G	-L*,+a*,-b* to +L*,-a*,+b*	
H	-L*,+a*,+b* to +L*,-a*,-b*	
I	-L*,-a*,+b* to +L*,+a*,-b*	

Table V: Color Centers Used in Phase II

Color Center	CIE recomd.	L *	a *	b *	ΔE^*_{ab} from Anchor	Also in Phase I
Anchor		49.21	0.045	5.275	-	yes
Moderate yellow	yes	77.2	2.0	36.0	41.6	yes
Grayish yellow green		64.6	-9.9	13.2	20.0	yes
Moderate bluish green	yes	55.0	-27.7	2.0	28.5	yes
Moderate blue	yes	34.2	-1.0	-28.0	36.5	yes
Grayish purple		45.6	11.4	-12.6	21.5	yes
Moderate greenish blue		49.1	-16.2	-11.5	23.4	yes
Dark redish orange	yes	42.8	34.7	22.8	39.4	yes
Light brown		61.2	13.2	20.0	23.1	yes
Medium Gray	yes	58.2	-0.3	0.8	10.0	yes
Light Gray		83.0	0.4	0.1	34.2	
Strong orange yellow		75.0	17.2	78.4	79.4	
Light bluish green		68.2	-30.2	-5.4	37.3	
Brilliant greenish blue		59.4	-13.1	-26.1	35.5	
Moderate purplish pink		67.6	31.2	-0.2	36.6	
Dark bluish green		31.3	-32.4	-5.4	38.6	
Dark blue		29.8	7.1	-31.0	41.7	
Moderate reddish brown		28.5	20.8	17.1	31.6	
Very dark red		17.3	24.2	3.4	40.1	
Black		14.3	-0.2	0.2	35.3	

Thirteen of the Phase I vectors were repeated by Phase II. Eight of these repeats were those which did not have high confidence metrics after Probit analysis. The other five repeats were associated with the Moderate Bluish Green color center. This latter set was included to insure the validity of pooling the data. The F, G, H, and I vector directions were probed by the Phase II experiment for all of the Phase I color centers. For all of the new Phase II color centers vector directions A, B, C, F, G, H, and I were investigated. Table V displays all of the color centers used in Phase II. The Phase II results were disappointing. They did not show the excellent statistics produced by Phase I. Almost 53% of the analyzed vectors showed low confidence metrics.

The Phase II experimental design utilized a method superior to that of Phase I for logging acceptance or rejection responses. Phase II rejections were denoted by crossing out test-pair sample numbers from a preprinted response form. Acceptances were passively noted by not having their numbers crossed out. This method worked well and did not produce the ambiguous situation experienced with the Phase I data.

After relogging the Phase II data from photocopies of the original response sheets several times, there still appeared to be discrepancies between the reported T50's in Table 4 of Reniff[3] and those calculated with the current data. Ms. Reniff kindly made available copies of tabulations from her laboratory notes[53] and these were compared with the current values. The comparisons appear in A-II.

Of the 642 test-pairs used in the Phase II study, 46 showed discrepancy between Reniff's and the current tabulations. Excepting test-pairs associated with the Dark Bluish Green B vector, the maximum magnitude of difference per test-pair was 2 rejections.

Only three vectors: Dark Bluish Green B, Strong Orange Yellow A, and Dark Blue I, had T50 values which deviated more than .04 ΔE^*_{ab} units between the two tallies. Not taking into account these exceptions, discussed below, the differences are considered minuscule and within the bounds of experimental error.

Dark Bluish Green direction B vector showed a systematic over counting by Reniff resulting in five extra rejections for each of the vector's five test-pairs. Strong Orange Yellow A and Dark Blue I were two of a total of three of the Phase II unfiltered vectors which appeared to have been poorly sampled by the test-pairs such that the Probit program extrapolated beyond the range of test-pair ΔE^*_{ab}'s to determine the T50 point. The statistics for these two vectors, as shown in Table A-II, lead to the conclusion that this extrapolation process is extremely sensitive to any small difference in tabulated results, and thus has dubious reliability. The example of Strong Orange Yellow direction A showed that a deviation of a single rejection for a single test-pair (sample #158) resulted in a T50 difference of 0.32 units. Dark Blue I displayed a difference of 0.10 T50 units for a single rejection change in three of its test-pairs. The third example of T50 extrapolation was Strong Orange Yellow direction C, but there were no differences in rejection frequencies.

Many of the vectors which did not display differences in tabulation for respective test-pairs did, however, show a difference in T50 value in the range of +-.01 units. The conclusion is that the disparity in these reported values are a result of differences in rounding technique between the researchers.

Only the current data were used for the present research.

H. This Thesis

The purpose of this work was to understand why the two previous experiments had not produced similar confidence statistics. Supplemental experiments were designed to study aspects of this question. An uncertainty analysis was performed on the two preexisting data sets and the newly derived data.

The conclusion which will be supported in the next chapter was that an increase in observer task difficulty was accompanied by a decrease in the precision with which individuals made color-difference comparison decisions. This, in itself, might not have affected the confidence statistics had the experimental design not called for individuals to make multiple observations per color vector. Since the Probit analysis procedure as used here did not discriminate between individual imprecision and observer to observer differential, the within-observer noise caused an improper increase in the calculated population variance. By attenuating within-observer fluctuation the between-observer statistics became more accurate.

A median filtering technique was used on the raw observer responses. The filtered responses were considered legitimate input to the Probit analysis following the assumption that CIELAB is an ordinal scale with respect to small color-differences and the fact that a median operation is known to "show invariantive" for transformations of ordinal scales.[27] Given that there is a monotonic relationship between human color-difference perception and CIELAB color-differences along lines in small regions of color-space, it would be true that the median of a set of observer responses for a single vector yields the same response as that given to the median ΔE^*_{ab} test-pair. The filter enforces this relationship for every three contiguous ΔE^*_{ab} ordered responses for each vector for each individual observer. The effect of the median filter was to reduce within-observer noise while preserving the true between-observer variance.

Berns, et al[4] included a preliminary report showing the results of filtering the pooled Phase I and Phase II data with the techniques developed for the present study. At the time of that report, the filtered data were believed to have been completely accurate. The subsequent relogging of the Phase II data has helped to create a more reliable data set. A corrected version of the first eight columns of the summarizing table from Berns can be found in Table XXI (see Chapter V). The Phase II data were updated according to the relogged data, whereas the Phase I data were no different from that previously reported.

When comparing Table XXI with the Berns table, only 51 of the 118* Phase II vectors showed a difference in T50 value. The maximum magnitude of change was 0.05 T50 units. This small difference would not have caused a qualitative difference in the article's several figures visualizing the 3-dimensional implications of the variations across color-space of derived T50 values.

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* 118 of the 119 Phase II vectors were used in the Berns table because one of the Phase II vectors repeated from Phase I had inferior precision when compared with its Phase I counterpart.

It is necessary to report an error in a Berns, et al[4] calculation in order to show how it has changed with respect to the current tabulations. When referring to the table which has now been superseded by Table XXI in this study, it was reported that 79% of the vectors passed the 0.05 χ^2 probability test. That value should have been 77%. The percentage raises to 79% for the current, relogged data.

III. Uncertainty Analysis

A. Approach

This thesis had the mandate to determine which of the differences between Phase I and Phase II caused the systematic increase in observer uncertainty. A list was made of all differences between Phases I and II. Phase I and II data were examined and where necessary supplemental data were gathered to help decide how each difference contributed to the change in confidence statistics:

- Different duration of observer sessions
- Different sample ΔE^*_{ab} range per vector
- Different observer population
- Different vector orientations
- Different color center distances from the anchor pair

Investigations of observer session duration and different sample ΔE^*_{ab} ranges per vector required supplemental data. Existing Phase I and II data as well as previous research were used to discuss the other differences.

B. Supplemental Observations

For the Supplemental observation sessions, two color centers from the previous Phases were chosen. Moderate Bluish Green was selected because it was the only fully repeated Phase I and Phase II color center. Light Bluish Green was the other selected color center. It had been used in Phase II, but had not been part of Phase I. Light Bluish Green had two characteristics which made it appropriate for the Supplemental experiment: 1) it was representative of those Phase II color centers for which observers had shown significant uncertainty as four out of its seven vectors were associated with low χ^2 probability terms; 2) with a color-distance of 37.3 ΔE^*_{ab} units from the anchor pair, it was typical of the Phase II color centers (see Table VI).

Table VI: Color Center Distance from Anchor Pair

All values in ΔE^*_{ab} units.

	Phase I Color Centers	Phase II Color Centers (Not including repeats)	Combined Phase I and Phase II Color Centers
Mean Color Center Distance from Anchor Pair	27.06	41.01	34.40
Median Color Center Distance from Anchor Pair	23.19	36.83	35.38
Maximum Color Center Distance from Anchor Pair	41.65	79.54	79.54
Minimum Color Center Distance from Anchor Pair	9.86	31.80	9.86

The Supplemental sessions were executed in a fashion similar to that of Phases I and II. Fifty color-normal volunteers, mostly consisting of RIT students, faculty and staff members were recruited for the 15 minute sessions. Observers sat in front of a MacBeth Spectralight booth and made observations under a daylight simulator with a correlated color temperature of 6250K. All other room lights were out. Observations consisted of comparing test-pairs of painted aluminum samples to the anchor pair. Observers were instructed to reject those test-pairs with color-differences exceeding that of the anchor pair. All others were accepted.

All colorimetric measurements for this study were made on a Milton Roy ColorScan 45 Spectrophotometer. Reniff[3] described the procedure for use of the instrument and how L*a*b* values were derived:

> The Milton Roy ColorScan 45/0 spectrophotometer was used for the measurements. The ColorScan is setup to run through the VAX/VMS system using software written at RIT. The instrument is a double beam scanning spectrophotometer which was calibrated using an NBS calibrated white porcelain tile. A program was written that prompted the user for measurement of both halves of the samples, columnized the data, and calculated the required data and color difference between the two. Each color difference pair had a measurement file containing ASCII data. The tristimulus calculation was [performed] using the ASTM weighting functions[56] for 10nm data, illuminant D65 and CIE 10° standard observer. A technique to correct the systematic errors in each measurement that was developed by Robertson[57] and modified by Berns and Petersen[58] was also used. An NBS calibrated cyan tile was measured and the systematic errors calculated from that measurement against the NBS values. Correction coefficients were then calculated and used for every measurement. New correction coefficients were calculated every half hour the instrument was in use in case there were any changes during its operation.

The anchor pair was originally constructed for the Phase I experiment. It consists of two near-neutral aluminum samples adhered to a gray mounting tile by double stick tape. It was also used by Phase II for consistency and again for these experiments.

Table VII: Anchor Pair Measurements and Calculations

	sample A	sample B	average
L*	49.53	48.89	49.21
a*	-0.08	0.17	0.045
b*	5.65	4.9	5.275

ΔE^*_{ab}
1.02

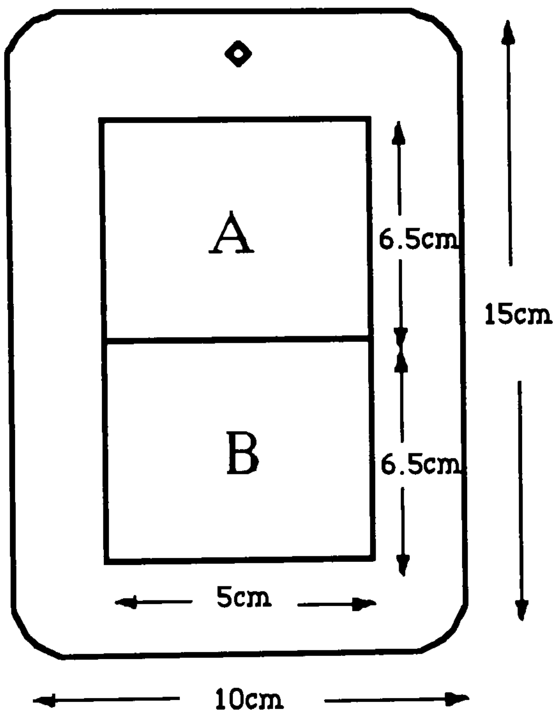

Figure 1: Anchor and Test-Pair Configuration[1]

Test-pairs were constructed to exactly the same dimensions as the anchor pair. They consisted of a pair of aluminum samples adhered to a gray mounting tile. Most test-pairs used for this study were constructed by Reniff for Phase II experiments. Several were constructed specifically for this study. For these newly constructed test-pairs, aluminum samples were chosen from a collection of painted samples which had been cut for Phase II purposes, but never used as part of a test-pair.

Tables VIII and IX compare the results from Phases I and II and the Supplemental observations.

Table VIII: Results for Moderate Bluish Green Color Center

Table VIIIa: Phase I

vector	T50	lower fl	upper fl	χ^2	χ^2 prob	stdev
A	0.945	0.905	1.008	4.213	0.519	0.31
B	2.283	2.209	2.363	3.171	0.787	0.453
C	1.316	1.239	1.366	4.095	0.536	0.397
D	1.672	1.623	1.736	3.104	0.684	0.346
E	1.774	1.718	1.831	1.168	0.948	0.398

Table VIIIb: Phase II

vector	T50	lower fl	upper fl	χ^2	χ^2 prob	stdev
A	1.033	0.936	1.18	2.797	0.424	0.572
B	2.579	2.412	2.841	0.139	0.987	0.972
C	1.224	1.138	1.306	1.636	0.651	0.475
D	1.782	1.651	1.919	5.535	0.137	0.776
E	1.812	1.682	1.935	3.502	0.478	0.808

Table VIIIc: Supplemental Observations

vector	T50	lower fl	upper fl	χ^2	χ^2 prob	stdev
A	1.188	0.849	*	14.15	0.003	0.529
B	2.251	1.833	2.771	8.248	0.041	0.694
C	1.138	1.069	1.201	2.257	0.521	0.345
D	1.493	1.331	1.616	0.513	0.916	0.763
E	1.449	1.324	1.541	5.582	0.233	0.492

*SAS Probit Program would not produce confidence limits due to high X^2

Table IX: Results for Light Bluish Green Color Center

Table IXa: Phase II

vector	T50	lower fl	upper fl	χ^2	χ^2 prob	stdev
A	1.202	*	*	30.32	0	1.178
B	2.375	2.186	2.539	2.868	0.413	0.951
C	1.781	1.673	1.892	2.422	0.49	0.618
F	1.293	1.207	1.375	6.928	0.14	0.459
G	1.694	1.328	2.01	11.95	0.008	0.393
H	1.709	*	*	26.6	0	1.228
I	1.475	0.813	1.784	12.52	0.006	0.436

Table IXb: Supplemental Observations

vector	T50	lower fl	upper fl	χ^2	χ^2 prob	stdev
A	1.586	0.343	3.199	45.7	1E-04	1
B	2.045	1.292	2.488	16.18	0.003	0.848
C	1.322	0.909	1.645	30.85	1E-04	0.476
F	1.295	0.897	1.691	39.92	1E-04	0.477
G	1.676	1.586	1.764	5.546	0.236	0.501
H	2.028	*	*	66.35	1E-04	0.884
I	1.261	0.875	1.509	29.39	1E-04	0.452

*SAS Probit Program would not produce confidence limits due to high X^2

C. Different Duration of Observer Session

Phase I had shorter observer sessions than Phase II. Phase I included 317 samples, Phase II had 636. Observers, thus, had to spend at least twice as long in the Phase II experiment. Fatigue and related problems had to be entertained as possible problems.

Samples associated with only the Moderate Bluish Green and the Light Bluish Green Color Centers, consisting of a total of 95 samples, were used for the Supplemental observations. The use of this relatively small sample set was designed to determine if reducing the time length of the observation session would significantly reduced the standard deviation of the observer responses. The task length was reduced to approximately 15 minutes, versus the three sessions of 45 minutes to an hour used in Phase II. Phase I had four sessions of 15-20 minute durations.

A t-test was performed to determine if, at 95% confidence level, the population responses had changed due to the shorter observational time.

Table X: T-test Comparison of Phase II and Supplemental Moderate Bluish Green and Light Bluish Green Standard Deviations

Phase II had three 45 60 minute sessions, Supplemental experiment had only one 15 minute session.

n = 12

	standard deviation mean	stdev	prob
Phase II	0.74	0.292	0.27
Supplemental	0.62	0.208	

The standard deviation of the combined Moderate Bluish Green and Light Bluish Green Color Centers were used to compare the experiments. At a 95% confidence level, probabilities of greater than 0.05 imply the two populations were the same.

Probabilities displayed in Table X show no significant difference between the Phase II and the Supplemental standard deviations despite the shorter session associated with the Supplemental task. This implies that the time duration of the Phase II task did not induce fatigue to the point that it affected the noisiness of the observations.

D. Different Sample ΔE^*_{ab} Range per Vector

The range of ΔE^*_{ab} units and the total number of sample pairs used per vector were smaller in the Phase II experiment than in Phase I. The Phase II experiment had been preceded by a pilot experiment which determined, among other things, the approximate location of the T50 for each vector. The Phase II Pilot had been performed in order to minimize the massive quantity of samples needed to be observed for the Phase II experiment.

In order to determine if the vector sampling had been improperly skewed by the Phase II pilot, the Supplemental experiment included an increased sampling of the Light Bluish Green color center. This was implemented by adding a relatively very small and very large color-difference along each vector.

A paired samples t-test was performed comparing the Supplemental T50's to the Phase II results along each Light Bluish Green vector to determine if the population answer changed significantly due to the increased sampling. A 95% confidence level was set.

Table XI: Paired Samples T-test Comparison of Phase II and Supplemental Light Bluish Green T50's

Supplemental had additional large and small color-difference test-pairs for each vector.

n = 7

	ΔT50		
	mean diff	stdev diff	prob
	0.04	0.32	0.73

At a 95% confidence level, probabilities of greater than 0.05 imply the two populations were the same. The high probability displayed in Table XI shows that the T50's did not vary significantly between the Phase II and Supplemental experiments. This indicates that the expected Phase II pilot T50 was a reasonable approximation of the actual T50 and that clustering around it did not malaffect the results.

E. Different Observer Population

A complete three way set of t-tests were performed to compare the populations from Phase I, Phase II and the Supplemental experiment as characterized by their Moderate Bluish Green responses.

Table XII: T-test Comparison of Phase I, II and Supplemental Moderate Bluish Green Responses

Moderate Bluish Green was the only color center fully repeated between all three Phases.

n = 5

	T50			χ^2 prob.			stdev		
	mean	stdev	prob	mean	stdev	prob	mean	stdev	prob
Phase I	1.6	0.5		0.70	0.18		0.38	0.05	
			0.81			0.36			0.02
Phase II	1.69	0.61		0.54	0.31		0.72	0.2	
Phase I	1.6	0.5		0.70	0.18		0.38	0.05	
			0.76			0.12			0.08
Supplemental	1.5	0.45		0.34	0.38		0.57	0.17	
Phase II	1.69	0.61		0.54	0.31		0.72	0.2	
			0.60			0.41			0.22
Supplemental	1.5	0.45		0.34	0.38		0.57	0.17	

At a 95% confidence level, probabilities of greater than 0.05 imply that the two populations were the same. From the probabilities presented in Table XII, the populations from the three experiments had similar T50's and χ^2 probabilities. The standard deviation comparison, as discussed below, shows a more complicated relationship.

T50 refers to the population color-difference tolerance. Table XII shows T50's to be statistically constant between the three experiments. χ^2 probability measures the degree to which the population responses followed a normal distribution. Table XII demonstrates that the populations from all three experiments were similarly normal.

The only significant differences between the experiments showed up in standard deviation. Standard deviation indicates the level of population precision. Table XII shows that with respect to standard deviation, the Phase I population was clearly different from the Phase II population. The Supplemental population standard deviation is not statistically different from Phase I or Phase II. The Phase I mean standard deviation for the Moderate Bluish Green color center was 0.38. This number increased to 0.57 for the Supplemental experiment and to 0.72 for Phase II.

Rich[59] has noted that visual scaling judgement precision quickly deteriorates with increasing task difficulty. Evidence cited below will indicate that the difficulty level of the Phase I, II and Supplemental tasks did correlate with respective standard deviations. The task difficulty will be shown to be a function of the vector directions in color-space and the color center distances from the anchor pair.

F. Different Vector Orientations

A comparison of Tables II and IV shows the differences between the Phase I and Phase II vector directions. Whereas all

Phase I vectors varied in either lightness or chromaticness, four of the Phase II vectors varied in both lightness and chromaticness.

Wyszecki[18] and Coates *et al.*[19] have reported an increase in difficulty for visual judgements made of samples which vary in both lightness and chromaticness compared to those which vary in only one or the other. Stroka *et al.*[20], however, found that combined lightness and chromaticness color-differences were not more difficult to judge.

Probit analysis includes two measures of observer uncertainty: the standard deviation and the χ^2 probability term. As a reminder, the standard deviation is a parameter defining the cumulative normal function which the Probit analysis has estimated to match the population response. The χ^2 probability term describes the degree to which the population has responded in a truly normal manner. Increased observer uncertainty would probably cause the standard deviation to rise and the χ^2 probability term to fall.

Table XIII compares the average χ^2 probability and the average standard deviation grouped by vector direction for Phase I and Phase II. It is interesting to note that in both Phase I and II, observers responded in a more non-normal fashion for pure lightness differences (vector directions A) than for those vectors which varied only in chromaticness (vector directions B, C, D and E). Even though the Phase I vector direction A average χ^2 probability term is relatively lower than the other vectors mentioned above, it is still statistically high enough to indicate that the Phase I population reacted normally to such color-differences. If it had been much lower, indicating a poor normality fit, the relatively low standard deviation for vector direction A might have been evidence that human lightness-difference response is non-normal with respect to ΔE^*_{ab}. Instead, the numbers indicate that the lightness judgements were most difficult for the Phase I population. Additionally, vector direction A contained the largest Phase II average standard deviation. The Phase I and II statistics agree that lightness judgements were more difficult than purely chromatic judgements. While this may be non-intuitive, due to the fact that human luminance vision has greater acuity than chromatic vision, it should be remembered that the color centers were primarily chromatic.[4]

Table XIII: Uncertainty Indicators for Phase I and II Grouped by Vector Direction

vector	CIELAB orientation	N	$\overline{\chi^2 \text{ prob}}$	$S_{\chi 2prob}$	$\overline{\text{stdev}}$	S_{stdev}
			Phase I			
A	-L* to +L*	9	0.274	0.287	0.353	0.051
B	-a* to +a*	9	0.421	0.324	0.378	0.059
C	-b* to +b*	9	0.323	0.355	0.431	0.081
D	-a*,-b* to +a*,+b*	9	0.521	0.335	0.407	0.147
E	-a*,+b* to +a*,-b*	9	0.648	0.366	0.449	0.182
			Phase II			
A	-L* to +L*	12	0.095	0.198	1.323	1.803
B	-a* to +a*	12	0.266	0.350	0.901	0.494
C	-b* to +b*	14	0.354	0.215	0.732	0.345
F	-L*,-a*,-b* to +L*,+a*,+b*	19	0.077	0.140	0.723	0.333
G	-L*,+a*,-b* to +L*,-a*,+b*	19	0.111	0.224	0.904	1.130
H	-L*,+a*,+b* to +L*,-a*,-b*	19	0.090	0.174	0.601	0.287
I	-L*,-a*,+b* to +L*,+a*,-b*	19	0.083	0.168	0.725	0.667

All Phase II average standard deviations were elevated with respect to Phase I average standard deviations. This indicates an increase in task difficulty. The χ^2 probability term data in Table XIII show that those vector directions which vary in lightness and chromaticness simultaneously (vectors F, G, H and I) were more difficult to judge than those which vary in chromaticness only. Duncan's multiple range test[63] verified that the χ^2 probabilities of the purely chromatic vector directions, B and C, were significantly different at a 95% confidence level from those which varied simultaneously in both dimensions. This evidence suggests that the Phase II task was more difficult in part due to the inclusion of vector directions that varied in both lightness and chromaticness.

G. Different Color Center Distance From Anchor Pair

Tables III, V, VI and Figure 2 illustrate the trend from Phase I to Phase II where average color center color-distance from the Anchor pair increased. Average Phase I color center was 27.06 ΔE^*_{ab} units from the anchor pair, average Phase II color center was 41.01 ΔE^*_{ab} units.

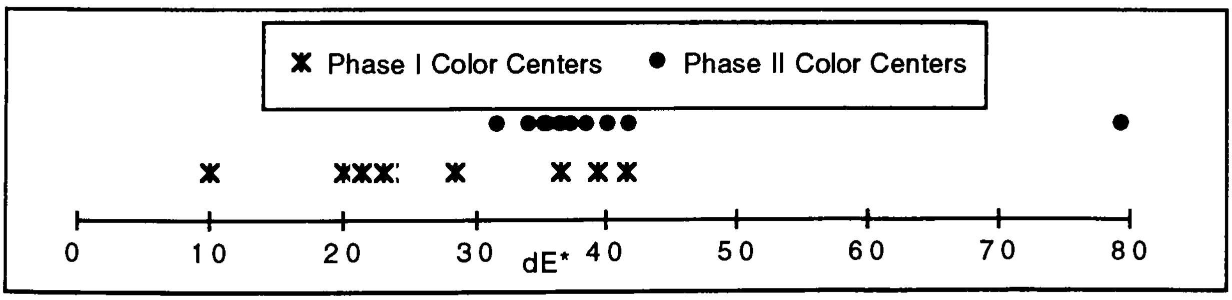

Figure 2: Phase I and Phase II Color Center ΔE^*_{ab} Distances from Anchor Pair

Table XIV: Uncertainty Indicators for Phase I Grouped by Color Center

Color Center	ΔE^*_{ab} from Anchor	$\overline{\chi^2 \text{ prob}}$	$S_{\chi 2prob}$	$\overline{\text{stdev}}$	S_{stdev}
Medium Gray	10	0.579	0.485	0.323	0.083
Grayish yellow green	20	0.424	0.341	0.349	0.049
Grayish purple	21.5	0.387	0.337	0.373	0.111
Light brown	23.1	0.544	0.383	0.348	0.093
Moderate greenish blue	23.4	0.247	0.371	0.486	0.113
Moderate bluish green	28.5	0.739	0.173	0.381	0.055
Moderate blue	36.5	0.263	0.39	0.519	0.172
Dark redish orange	39.4	0.298	0.178	0.459	0.127
Moderate yellow	41.6	0.828	0.134	0.394	0.099

Table XV: Uncertainty Indicators for Phase II Grouped by Color Center

Color Center	ΔE^*_{ab} from Anchor	$\overline{\chi^2 \text{ prob}}$	$S_{\chi 2prob}$	$\overline{\text{stdev}}$	S_{stdev}
Moderate reddish brown	31.6	0.141	0.214	0.696	0.339
Light gray	34.2	0.368	0.277	0.661	0.171
Black	35.3	0.1	0.116	0.574	0.143
Brilliant greenish blue	35.5	0.059	0.144	0.892	0.415
Moderate purplish pink	36.6	0.123	0.263	0.731	0.186
Light bluish green	37.3	0.151	0.212	0.752	0.361
Dark bluish green	38.6	0.077	0.139	0.983	0.509
Very dark red	40.1	0.141	0.26	0.706	0.156
Dark blue	41.7	0.099	0.22	0.995	1.02
Strong orange yellow	79.4	0.129	0.174	2.069	2.196

Stroka[20] found that color-distance judgement precision decreased as color center location moved further from the anchor pair. A similar trend is clear from examination of the average standard deviation values in Table XIV and Table XV and illustrated in Figure 3. A correlation analysis of the relationship between ΔE^*_{ab} distance from anchor pair and the average standard deviation for Phase II color centers showed a r^2 coefficient of determination of 0.931. A correlation analysis of the relationship between color center distance from the anchor pair and average standard deviations across pooled Phase I and Phase II color centers revealed a r^2 coefficient of determination of 0.768 (see Figure 3). These statistics lead to the conclusion that the increase in color center distance from the anchor pair contributed to a more difficult Phase II task.

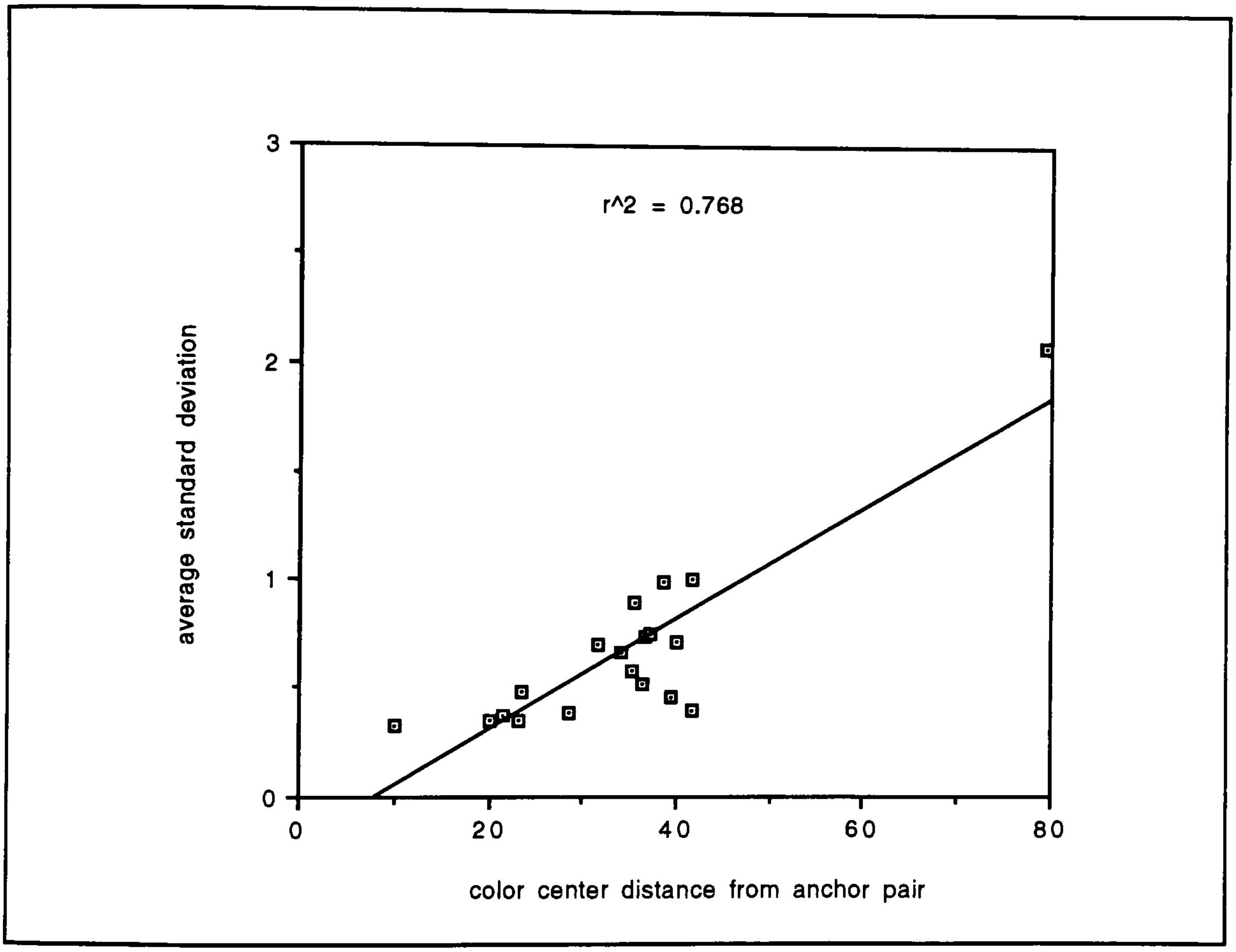

Figure 3: Average Stdev as Function of Color Center Color-Distance from Anchor Pair for Combined Phase I and II

IV. Median Filtering

A. Intra-Observer Filtering

As was shown by previous explanations, observer uncertainty increased from Phase I to Phase II because of task difficulty. Task difficulty was affected by vector directions which varied in both lightness and chromaticness simultaneously, and by color center distance from the anchor pair.

Probit analysis was designed to deal well with inter-observer variance. Generally, Probit has been used in experiments where subjects give only one response, thus, there is no precedent in the literature of how to treat noisy individual observer responses. The fact that Phase I and II observers gave multiple answers per color center vector means that intra-observer noise was included in each Probit data set.

Recall that test pairs with perceived color-difference magnitude smaller than the anchor pair were accepted, those with color-difference magnitude larger than the anchor pair were rejected. Assuming local monotonicity of CIELAB with respect to color-difference perception, the responses for a perfectly noise free observer for a single vector when ordered in ascending ΔE^*_{ab} would show a series of accepted test pairs up to his tolerance level and all test pairs with larger CIELAB color-differences would be rejected. Figure 4 demonstrates how such an observer would react to a fictional color center and vector.

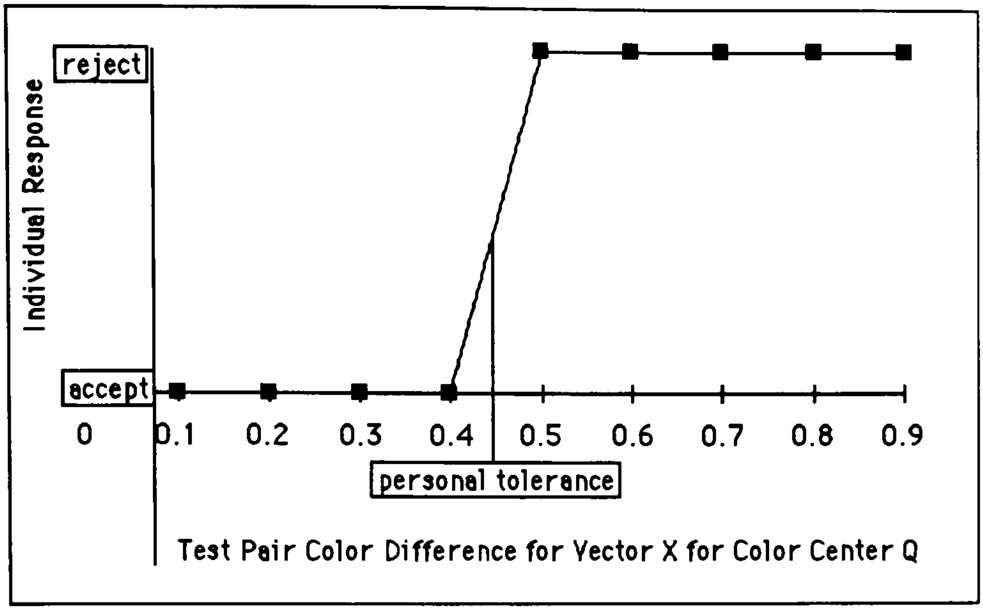

Figure 4: Perfectly Noise Free Observer for a Single Vector (Example I)

A set of these perfectly noise free observers would not necessarily agree on where the break point was between acceptable test pairs and rejected test pairs. As in Figure 5, this lack of agreement would define the level of inter-observer variability.

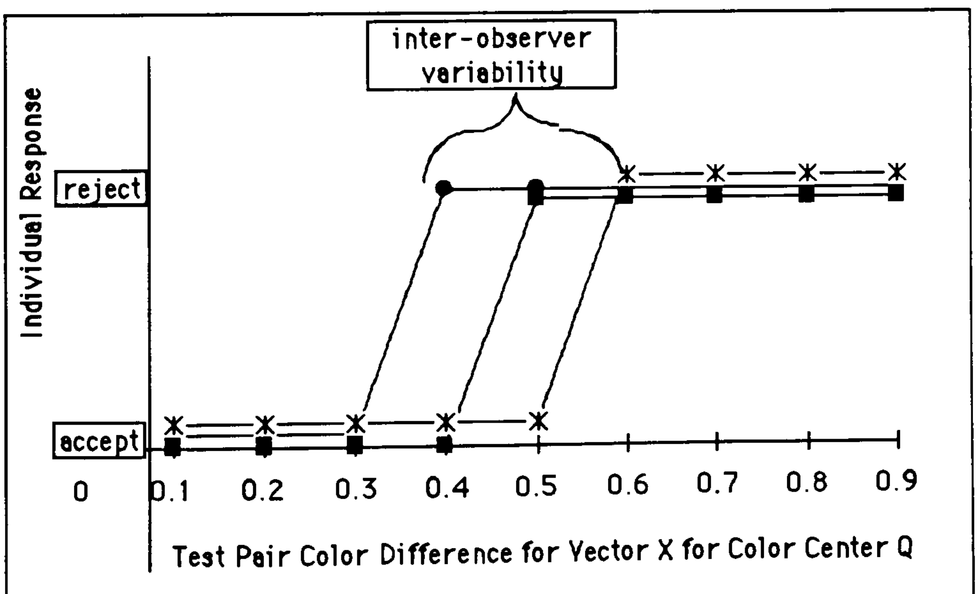

Figure 5: Set of Noise Free Observers for a Single Vector (Example I)

Table XVI shows the data which would be used for the Probit analysis given the data from Figure 5.

Table XVI: Frequency Response for Noise Free Observers (Example I)

ΔE^*_{ab}	# responding
0.1	0
0.2	0
0.3	0
0.4	1
0.5	2
0.6	3
0.7	3
0.8	3
0.9	3

Within-observer variability manifested in individuals responding as if CIELAB were non-monotonic locally: a rejected test pair had a smaller ΔE^*_{ab} than an accepted test pair belonging to the same color center vector, see Figure 6. Since Probit does not distinguish between multiple responses from single observers and single responses from multiple observers, this intra-observer noise was treated by the analysis as if it were additional inter-observer variance, see Figure 7 and Table XVII.

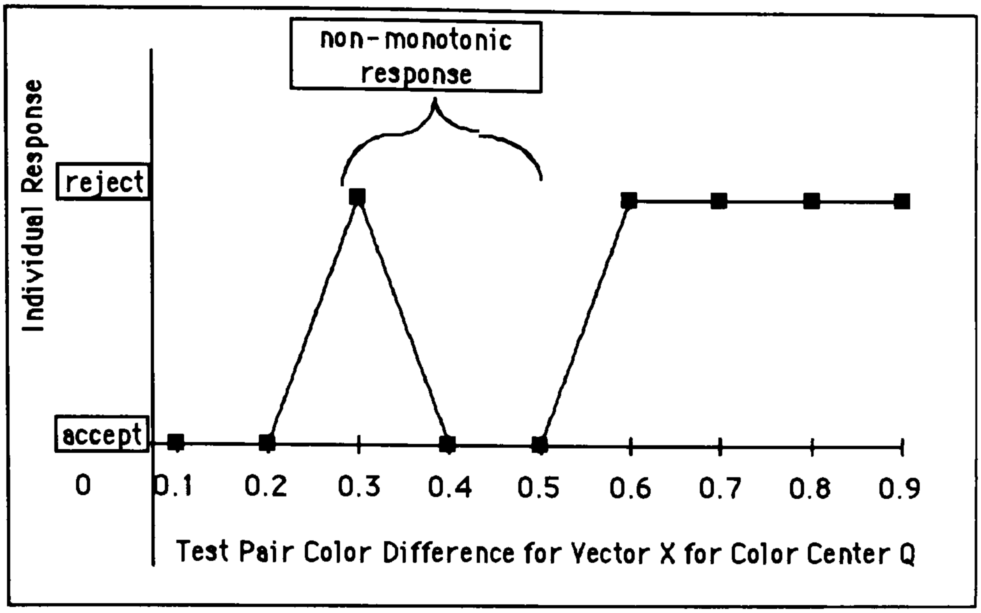

Figure 6: Individual Responding as if CIELAB were Non-Monotonic Locally (Example II)

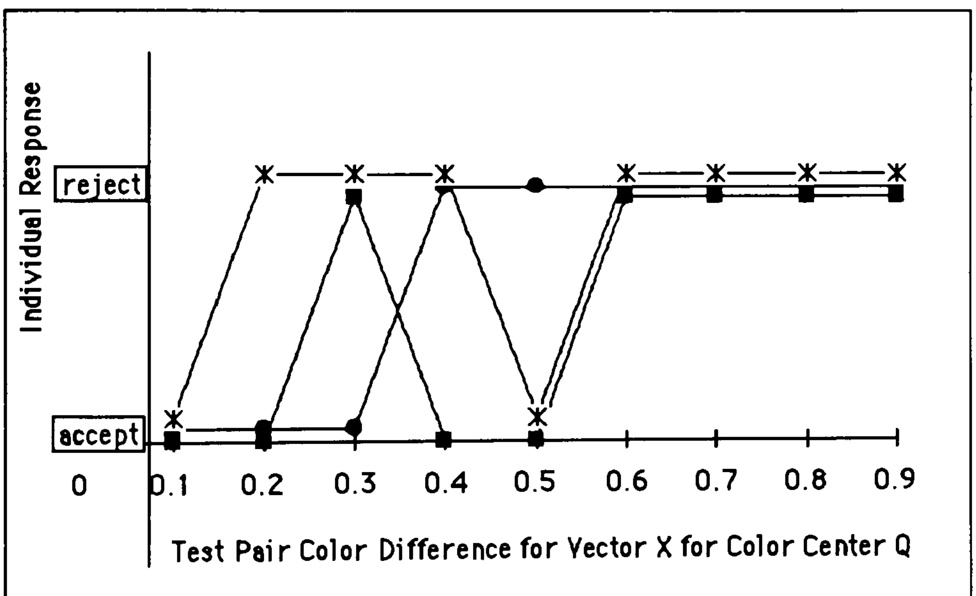

Figure 7: Set of Observers Responding as if CIELAB were Non-Monotonic Locally (Example II)

Table XVII: Frequency Response for Noisy Observers (Example II)

ΔE^*_{ab}	# responding
0.1	0
0.2	1
0.3	2
0.4	2
0.5	1
0.6	3
0.7	3
0.8	3
0.9	3

There are many instances cited in the literature where raw observer responses are responsibly filtered prior to tallying ensemble statistics.[6,64-66] Use of *a priori* knowledge is supported by Worthing[6] as being legitimate for adjusting measurements so they might better reflect laws which relate to the quantities. One law used by this study to rationalize within-observer filtering is the above mentioned assumption of local CIELAB monotonicity with respect to color-difference. Although these studies are challenging the effectiveness of color-difference scales such as CIELAB as *ratio scales*, there is no question of CIELAB's legitimacy as an *ordinal scale* for small color-differences. Thus, responses such as those given by the above described noisy observers are violations of an accepted law, that of the order-preserving nature of local CIELAB color-differences, and may be conditioned to improve consistency with the law.

The median is known to "show invariantive" for transformations of ordinal scales.[27] Imposing this relationship on the observer responses would remove behavior which violated the ordinal nature of local CIELAB color-differences. A three-wide median filter algorithm was used on ordered observer vector responses. Median filters have been used by electrical engineers and image processing professionals for removing narrow peaks and valleys from signals. The width of a median filter kernel defines the width of peaks and valleys affected by its application (see Figure 8). In the case of a 1-dimensional bi-level quantized signal, a median filter kernel with a width of three sample points will only change those signal segments with structures as narrow as a single sample point. Thus, a three-wide median filter makes modifications only in the special situation where the signal oscillates between the binary levels, say 0 and 1, in a 1-0-1 or 0-1-0 configuration. In the example of a 1-0-1 signal segment, the three-wide median filter would output 1 for the center value. For 0-1-0 , the median filter would produce a 0 for the center value. The filtering would not change the center value for any other three-wide signal segment. Analogies were made corresponding observer responses to signal levels (0 for accept, 1 for reject), and signals to sets of individual observer responses for each vector, ordered according to ascending ΔE^*_{ab}. These analogies helped to envision a three-wide median filter used to remove a level of intra-observer self-contradictory behavior which can be thought of as high frequency noise.

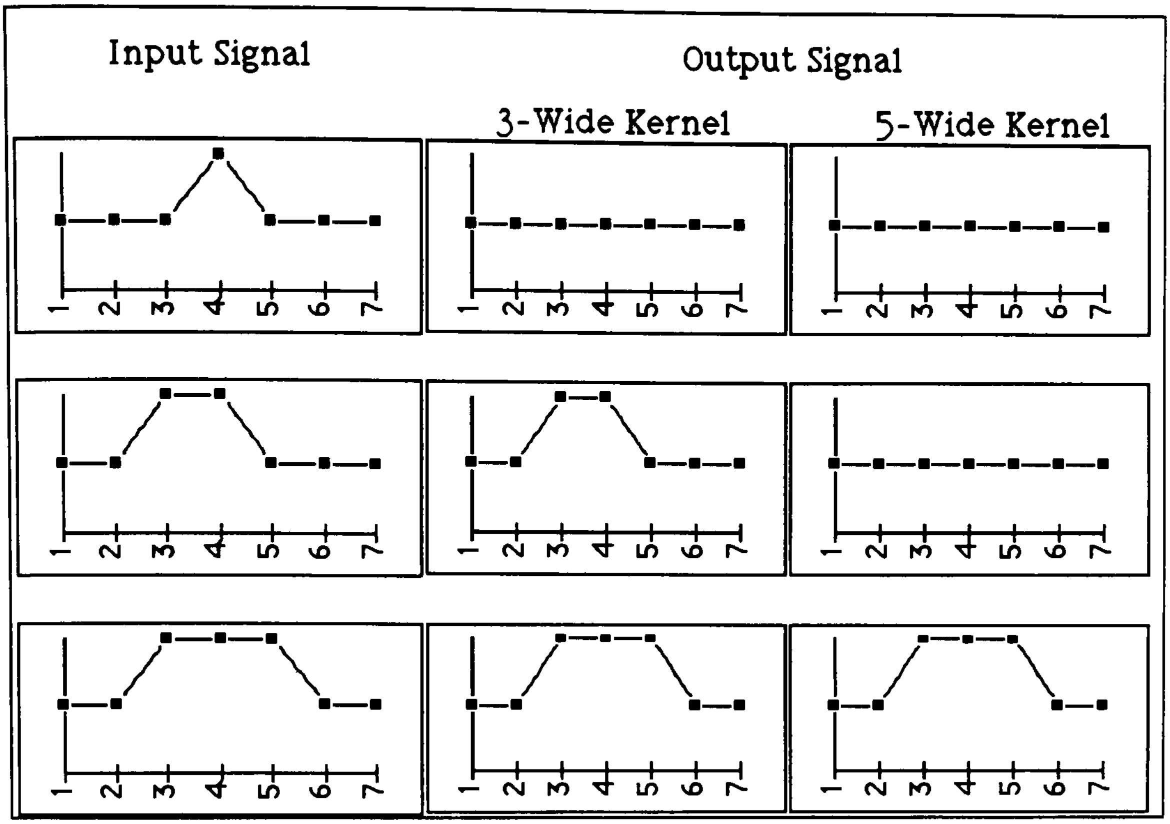

Figure 8: Median Filter Examples

It should be noted that not all observer self-contradictory behavior was removed by the median filtering with a three-wide kernel. As explained above, any structure wider than a single sample point, as contained in the example 0-0-1-1-0-0, would have been untouched. Also, a completely confused observer response signal, such as 1-0-1-0-1-0, would be changed, in this case to 0-1-0-1-0-1, thus continuing to exhibit its oscillatory characteristic.

Table XVIII compares the filtered and unfiltered Phase I Probit input and the T50 results. Table XIX shows the same type of data for Phase II.

Table XVIII: Comparing Phase I Unfiltered to Filtered Frequency Data

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Moderate blue	A						0.96	0.96	0.00
		1	0.26	0	0	0			
		2	0.5	10	7	-3			
		3	0.75	16	18	2			
		4	1.01	28	28	0			
		5	1.22	39	41	2			
		6	1.69	47	47	0			
		7	1.99	48	48	0			
	B						1.36	1.37	0.01
		8	0.53	1	0	-1			
		9	0.82	5	1	-4			
		10	1.14	16	20	4			
		11	1.4	31	27	-4			
		12	1.75	40	43	3			
		13	1.96	46	46	0			
		14	2.35	47	48	1			
	C						1.54	1.55	0.01
		15	0.48	0	0	0			
		16	1.19	14	10	-4			
		17	1.47	22	22	0			
		18	1.74	31	30	-1			
		19	1.96	41	44	3			
		20	2.21	47	48	1			
		21	2.45	49	50	1			
	D						1.12	1.12	0.00
		22	0.34	0	0	0			
		23	0.42	3	2	-1			
		24	0.76	5	5	0			
		25	1.09	22	22	0			
		26	1.22	39	38	-1			
		27	1.49	41	42	1			
		28	1.89	47	48	1			
	E						2.83	2.84	0.01
		29	1.47	1	0	-1			
		30	1.99	8	8	0			
		31	2.56	14	11	-3			
		32	2.83	28	24	-4			
		33	3.06	33	36	3			
		34	3.48	41	44	3			
		35	3.67	45	45	0			
		36	4.73	47	48	1			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Moderate greenish blue	A						0.79	0.78	-0.01
		37	0.25	3	2	-1			
		38	0.5	7	7	0			
		39	0.75	25	25	0			
		40	0.97	36	40	4			
		41	1.21	47	45	-2			
		42	1.56	47	49	2			
		43	1.8	50	50	0			
	B						1.61	1.62	0.01
		44	0.72	1	1	0			
		45	0.96	3	2	-1			
		46	1.43	14	13	-1			
		47	1.8	37	34	-3			
		48	2.05	41	44	3			
		49	2.26	48	47	-1			
		50	2.33	48	50	2			
	C						1.63	1.62	-0.01
		51	0.76	0	0	0			
		52	1.21	10	6	-4			
		53	1.47	17	17	0			
		54	1.75	27	30	3			
		55	1.95	46	46	0			
		56	2.44	48	49	1			
		57	2.95	48	49	1			
	D						1.8	1.8	0.00
		58	1	3	3	0			
		59	1.35	13	10	-3			
		60	1.56	17	17	0			
		61	1.85	22	24	2			
		62	2.12	44	43	-1			
		63	2.62	44	46	2			
		64	3.18	48	48	0			
	E						1.5	1.48	-0.02
		65	0.73	3	3	0			
		66	0.98	8	7	-1			
		67	1.57	33	29	-4			
		68	1.93	38	37	-1			
		69	2.32	46	48	2			
		70	3.06	49	49	0			
		71	3.12	50	50	0			
Medium gray	A						0.93	0.94	0.01
		72	0.24	1	0	-1			
		73	0.48	2	1	-1			
		74	0.72	9	7	-2			
		75	0.96	27	27	0			
		76	1.29	46	48	2			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		77	1.72	50	50	0			
		78	1.99	50	50	0			
	B						0.89	0.9	0.01
		79	0.24	0	0	0			
		80	0.52	5	1	-4			
		81	0.76	15	16	1			
		82	1	37	38	1			
		83	1.21	44	45	1			
		84	1.61	49	49	0			
		85	1.83	49	49	0			
	C						1.33	1.32	-0.01
		86	0.74	2	2	0			
		87	1.01	17	16	-1			
		88	1.56	33	33	0			
		89	2.09	47	49	2			
		90	2.17	49	49	0			
		91	2.54	50	50	0			
		92	3.08	50	50	0			
	D						0.92	0.93	0.01
		93	0.48	1	0	-1			
		94	0.71	9	8	-1			
		95	0.96	30	30	0			
		96	1.2	43	45	2			
		97	1.43	49	49	0			
		98	1.66	50	50	0			
		99	2.19	50	50	0			
	E						1.3	1.27	-0.03
		100	0.9	6	6	0			
		101	1.18	19	17	-2			
		102	1.4	32	33	1			
		103	1.91	47	50	3			
		104	2.29	50	50	0			
		105	2.43	50	50	0			
		106	2.81	50	50	0			
Moderate bluish green	A						0.94	0.96	0.02
		107	0.3	2	0	-2			
		108	0.53	4	2	-2			
		109	0.74	10	9	-1			
		110	0.98	30	30	0			
		111	1.26	40	43	3			
		112	1.47	49	49	0			
		113	2.03	50	50	0			
	B						2.28	2.28	0.00
		114	0.82	0	0	0			
		115	1.49	1	0	-1			
		116	1.69	4	4	0			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		117	1.96	16	13	-3			
		118	2.2	22	21	-1			
		119	2.41	28	32	4			
		120	2.93	46	46	0			
		121	3.68	50	50	0			
	C						1.32	1.3	-0.02
		122	0.56	0	0	0			
		123	0.78	4	3	-1			
		124	1.04	14	13	-1			
		125	1.24	24	24	0			
		126	1.51	35	35	0			
		127	1.81	42	46	4			
		128	2.09	49	49	0			
	D						1.67	1.68	0.01
		129	0.57	0	0	0			
		130	0.98	1	0	-1			
		131	1.25	5	3	-2			
		132	1.46	17	13	-4			
		133	1.71	24	29	5			
		134	2	40	40	0			
		135	2.13	47	48	1			
	E						1.77	1.77	0.00
		136	1.06	2	1	-1			
		137	1.22	4	3	-1			
		138	1.53	13	9	-4			
		139	1.75	22	20	-2			
		140	1.93	35	35	0			
		141	2.15	42	46	4			
		142	2.54	48	50	2			
Light brown	A						0.93	0.9	-0.03
		143	0.27	2	1	-1			
		144	0.49	2	3	1			
		145	0.73	17	10	-7			
		146	0.96	23	29	6			
		147	1.26	47	47	0			
		148	1.53	48	50	2			
		149	1.93	49	50	1			
	B						1.39	1.39	0.00
		150	0.47	0	0	0			
		151	0.78	1	1	0			
		152	1.24	16	14	-2			
		153	1.53	38	35	-3			
		154	1.83	43	48	5			
		155	2.03	48	49	1			
		156	2.31	50	50	0			
	C						1.46	1.48	0.02
		157	0.54	2	2	0			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		158	0.98	5	2	-3			
		159	1.33	14	11	-3			
		160	1.5	25	29	4			
		161	1.83	45	43	-2			
		162	2.01	46	48	2			
	D						1.6	1.59	-0.01
		163	0.7	1	0	-1			
		164	0.96	2	1	-1			
		165	1.22	14	14	0			
		166	1.75	33	34	1			
		167	1.99	39	39	0			
		168	2.25	45	47	2			
		169	3.03	50	50	0			
	E						1.13	1.14	0.01
		170	0.24	0	0	0			
		171	0.86	5	3	-2			
		172	1.08	19	17	-2			
		173	1.25	37	39	2			
		174	1.45	46	47	1			
		175	2.01	50	50	0			
		176	2.21	50	50	0			
Grayish purple	A						0.96	0.94	-0.02
		177	0.24	0	0	0			
		178	0.75	15	14	-1			
		179	1.23	42	41	-1			
		180	1.49	46	49	3			
		181	1.7	48	50	2			
		182	1.99	50	50	0			
	B						1.47	1.47	0.00
		183	0.54	0	0	0			
		184	1.03	2	2	0			
		185	1.28	16	14	-2			
		186	1.48	31	32	1			
		187	1.82	41	41	0			
		188	2.06	46	48	2			
		189	2.26	50	50	0			
	C						1.42	1.42	0.00
		190	0.74	0	0	0			
		191	1.04	3	3	0			
		192	1.21	9	8	-1			
		193	1.44	36	36	0			
		194	1.82	45	45	0			
		195	1.99	47	48	1			
	D						1.23	1.23	0.00
		196	0.7	2	2	0			
		197	0.95	8	6	-2			
		198	1.14	22	22	0			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		199	1.42	37	39	2			
		200	1.78	48	48	0			
		201	2.01	49	49	0			
	E						2.87	2.88	0.01
		202	1.25	0	0	0			
		203	1.49	0	0	0			
		204	1.73	2	0	-2			
		205	2.02	2	2	0			
		206	2.23	4	4	0			
		207	2.44	14	13	-1			
		208	2.85	25	25	0			
		209	3.04	33	33	0			
		210	3.91	47	47	0			
Dark redish orange	A						0.95	0.94	-0.01
		211	0.35	2	1	-1			
		212	0.49	4	5	1			
		213	0.76	18	12	-6			
		214	1	25	30	5			
		215	1.26	44	45	1			
		216	1.52	47	47	0			
		217	2.01	49	50	1			
	B						1.95	1.94	-0.01
		218	0.75	0	0	0			
		219	1.26	1	0	-1			
		220	1.5	5	5	0			
		221	1.75	15	12	-3			
		222	2.02	24	27	3			
		223	2.15	42	44	2			
		224	2.49	46	47	1			
		225	2.97	50	50	0			
	C						1.54	1.54	0.00
		226	0.5	0	0	0			
		227	0.99	4	4	0			
		228	1.24	20	17	-3			
		229	1.49	26	27	1			
		230	1.8	29	31	2			
		231	2	43	42	-1			
		232	2.54	48	49	1			
	D						2.02	2.01	-0.01
		233	1.14	3	1	-2			
		234	1.47	9	8	-1			
		235	1.79	18	19	1			
		236	1.98	27	28	1			
		237	2.23	37	35	-2			
		238	2.54	36	40	4			
		239	2.96	45	45	0			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
	E						1.32	1.32	0.00
		240	0.26	1	1	0			
		241	1.02	12	4	-8			
		242	1.23	16	22	6			
		243	1.51	37	38	1			
		244	1.77	43	44	1			
		245	2.04	48	49	1			
		246	2.56	50	50	0			
Moderate yellow	A						1.18	1.19	0.01
		247	0.46	2	1	-1			
		248	0.74	6	3	-3			
		249	0.98	11	11	0			
		250	1.25	29	32	3			
		251	1.48	42	41	-1			
		252	1.78	46	47	1			
		253	2.08	50	50	0			
	B						1.45	1.44	-0.01
		254	0.84	2	1	-1			
		255	0.91	4	3	-1			
		256	1.22	13	11	-2			
		257	1.52	27	28	1			
		258	1.74	40	42	2			
		259	2.03	47	49	2			
		260	2.39	49	50	1			
	C						2.21	2.2	-0.01
		261	0.76	0	0	0			
		262	1.03	0	0	0			
		263	1.22	2	2	0			
		264	1.75	11	7	-4			
		265	2.08	23	20	-3			
		266	2.28	25	30	5			
		267	2.5	36	37	1			
		268	2.84	44	46	2			
		269	3.17	47	48	1			
	D						1.63	1.63	0.00
		270	0.82	1	0	-1			
		271	1.04	4	1	-3			
		272	1.28	6	6	0			
		273	1.5	16	16	0			
		274	1.72	32	33	1			
		275	2.08	45	46	1			
	E						1.29	1.28	-0.01
		276	0.72	1	0	-1			
		277	0.95	6	4	-2			
		278	1.17	16	17	1			
		279	1.44	36	39	3			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		280	1.74	46	48	2			
		281	1.94	50	50	0			
		282	2.64	50	50	0			
Grayish yellow green	A						0.86	0.86	0.00
		283	0.28	2	1	-1			
		284	0.47	3	3	0			
		285	0.74	24	21	-3			
		286	1	35	37	2			
		287	1.26	43	46	3			
		288	1.52	48	48	0			
		289	2	49	49	0			
	B						1.16	1.17	0.01
		290	0.5	2	0	-2			
		291	0.8	4	3	-1			
		292	1.02	19	17	-2			
		293	1.29	34	34	0			
		294	1.5	42	45	3			
		295	1.68	49	49	0			
		296	1.96	48	49	1			
	C						1.44	1.44	0.00
		297	0.66	1	1	0			
		298	1.02	9	7	-2			
		299	1.27	14	13	-1			
		300	1.54	26	26	0			
		301	1.76	39	43	4			
		302	1.98	50	50	0			
		303	2.42	50	50	0			
	D						1.21	1.21	0.00
		304	0.71	1	1	0			
		305	0.99	13	8	-5			
		306	1.18	21	24	3			
		307	1.51	45	44	-1			
		308	1.76	48	49	1			
		309	1.95	50	50	0			
		310	2.17	50	50	0			
	E						1.69	1.72	0.03
		311	0.82	1	0	-1			
		312	1.51	13	12	-1			
		313	1.77	31	27	-4			
		314	1.96	39	43	4			
		315	2.37	48	49	1			
		316	2.53	48	49	1			
		317	3.25	50	50	0			

Table XVIII (Cont.)

Color Center	Vector Set	Snyder Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Statistics									
Number of Cases						317	45	45	45
Maximum						6	2.87	2.88	0.03
Minimum						-8	0.79	0.78	-0.03
Mean						0.012	1.441	1.440	0
Stdev						1.950	0.469	0.471	0.012

Table XIX: Comparing Phase II Unfiltered to Filtered Frequency Data

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Light bluish green	A						1.20	1.26	0.06
		1	1.01	8	8	0			
		2	1.19	36	31	5			
		3	1.53	37	40	-3			
		4	2.09	38	38	0			
		5	2.45	40	42	-2			
	B						2.38	2.38	0.00
		6	1.90	17	9	8			
		7	2.23	20	21	-1			
		8	2.50	25	30	-5			
		9	3.00	41	43	-2			
		10	3.99	47	49	-2			
	C						1.78	1.78	0.00
		11	1.47	15	2	13			
		12	1.17	7	10	-3			
		13	1.69	22	23	-1			
		14	2.00	36	37	-1			
		15	2.51	42	47	-5			
	F						1.29	1.31	0.02
		16	0.59	4	2	2			
		17	1.05	9	7	2			
		18	1.36	30	27	3			
		19	1.45	36	41	-5			
		20	1.95	46	47	-1			
		21	2.43	49	49	0			
	G						1.69	1.68	-0.01
		22	1.24	3	3	0			
		23	1.56	16	11	5			
		24	1.67	34	33	1			
		25	2.01	37	43	-6			
		26	2.38	47	49	-2			
	H						1.71	1.67	-0.04
		27	1.29	17	8	9			
		28	1.63	13	22	-9			
		29	1.74	40	30	10			
		30	1.98	29	42	-13			
		31	2.57	36	47	-11			
	I						1.47	1.47	0.00
		32	1.15	11	5	6			
		33	1.30	16	20	-4			
		34	1.68	42	34	8			
		35	1.82	32	44	-12			
		36	1.98	45	48	-3			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Brilliant greenish blue	A						1.24	1.23	-0.01
		37	0.47	4	1	3			
		38	0.76	25	22	3			
		39	0.95	29	25	4			
		40	1.27	20	22	-2			
		41	1.60	23	30	-7			
		42	1.88	41	41	0			
	B						1.51	1.67	0.16
		43	1.26	17	9	8			
		44	1.68	32	30	2			
		45	1.78	35	32	3			
		46	2.15	22	35	-13			
		47	2.23	37	42	-5			
	C						2.05	2.07	0.02
		48	1.42	11	4	7			
		49	1.70	16	16	0			
		50	2.18	31	34	-3			
		51	2.60	39	37	2			
		52	2.82	37	44	-7			
	F						1.02	1.00	-0.02
		53	0.76	7	4	3			
		54	0.99	34	35	-1			
		55	1.43	47	44	3			
		56	1.69	33	49	-16			
		57	2.09	50	50	0			
	G						1.62	1.67	0.05
		58	1.30	18	6	12			
		59	1.56	16	25	-9			
		60	1.72	35	31	4			
		61	2.04	34	39	-5			
		62	2.64	46	47	-1			
	H						1.37	1.36	-0.01
		63	0.98	8	7	1			
		64	1.37	31	20	11			
		65	1.40	16	30	-14			
		66	1.66	40	34	6			
		67	1.56	38	45	-7			
		68	2.37	48	49	-1			
	I						2.07	2.19	0.12
		69	1.74	23	11	12			
		70	2.12	24	18	6			
		71	2.24	17	27	-10			
		72	2.46	36	34	2			
		73	2.64	46	43	3			
		74	2.89	41	48	-7			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Moderate purplish pink	A						1.34	1.34	0.00
		75	0.89	4	3	1			
		76	1.04	20	13	7			
		77	1.24	28	28	0			
		78	1.56	29	32	-3			
		79	1.72	35	41	-6			
		80	1.87	43	43	0			
	B						2.41	2.41	0.00
		81	1.47	7	1	6			
		82	1.94	12	13	-1			
		83	2.26	18	21	-3			
		84	2.38	36	27	9			
		85	2.62	27	33	-6			
		86	2.94	34	36	-2			
	C						1.94	1.91	-0.03
		87	1.45	14	10	4			
		88	1.82	23	20	3			
		89	2.04	25	24	1			
		90	2.11	28	37	-9			
		91	2.53	41	46	-5			
	F						1.70	1.68	-0.02
		92	1.15	21	8	13			
		93	1.30	12	17	-5			
		94	1.59	26	14	12			
		95	1.76	15	26	-11			
		96	2.01	38	41	-3			
	G						1.58	1.58	0.00
		97	0.50	2	0	2			
		98	0.62	1	1	0			
		99	0.83	8	6	2			
		100	0.97	5	10	-5			
		101	0.96	24	8	16			
		102	1.51	13	15	-2			
		103	1.75	34	35	-1			
	H						1.59	1.58	-0.01
		104	0.82	5	2	3			
		105	1.18	4	5	-1			
		106	1.33	18	14	4			
		107	1.62	28	28	0			
		108	1.92	37	40	-3			
		109	2.15	43	47	-4			
	I						1.72	1.69	-0.03
		110	0.89	2	1	1			
		111	1.33	13	11	2			
		112	1.46	19	19	0			
		113	1.82	36	34	2			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		114	2.37	37	43	-6			
Light gray	A						1.26	1.30	0.04
		115	1.09	16	15	1			
		116	1.31	34	27	7			
		117	1.54	29	37	-8			
		118	1.96	39	42	-3			
		119	2.14	44	43	1			
		120	2.30	42	47	-5			
	B						0.99	1.02	0.03
		121	0.96	22	18	4			
		122	1.09	30	30	0			
		123	1.23	30	36	-6			
		124	1.49	42	42	0			
		125	2.04	46	48	-2			
	C						1.80	1.83	0.03
		126	1.34	11	9	2			
		127	1.81	29	23	6			
		128	2.11	29	33	-4			
		129	2.17	38	42	-4			
		130	2.78	47	46	1			
		131	2.81	47	48	-1			
	F						1.24	1.26	0.02
		132	0.88	9	4	5			
		133	0.95	20	17	3			
		134	1.20	27	25	2			
		135	1.41	30	31	-1			
		136	1.72	34	39	-5			
		137	1.86	45	48	-3			
	G						1.36	1.36	0.00
		138	0.95	13	9	4			
		139	1.26	22	22	0			
		140	1.38	28	28	0			
		141	1.62	32	34	-2			
		142	1.88	37	42	-5			
	H						1.02	1.01	-0.01
		143	0.52	4	3	1			
		144	0.82	14	8	6			
		145	1.05	21	25	-4			
		146	1.19	38	44	-6			
		147	1.66	49	49	0			
	I						1.56	1.55	-0.01
		148	0.93	11	7	4			
		149	1.40	19	19	0			
		150	1.58	27	23	4			
		151	1.84	29	35	-6			
		152	2.01	39	43	-4			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Strong orange yellow	A						0.34	1.61	1.27
		153	1.42	23	17	6			
		154	1.54	32	25	7			
		155	1.75	27	30	-3			
		156	1.84	32	33	-1			
		157	2.03	36	33	3			
		158	2.28	25	41	-16			
	B						1.80	1.88	0.08
		159	1.39	21	13	8			
		160	1.68	22	20	2			
		161	2.08	25	26	-1			
		162	2.34	31	35	-4			
		163	2.65	46	49	-3			
	C						5.13	4.46	-0.67
		164	2.79	7	3	4			
		165	3.17	6	6	0			
		166	3.32	7	6	1			
		167	3.55	6	6	0			
		168	3.78	13	8	5			
		169	3.98	15	21	-6			
	F						1.97	2.10	0.13
		170	1.62	17	14	3			
		171	1.85	31	21	10			
		172	2.05	22	26	-4			
		173	2.42	30	28	2			
		174	2.85	38	42	-4			
		175	3.76	46	47	-1			
	G						2.14	2.06	-0.08
		176	1.35	5	2	3			
		177	1.41	16	17	-1			
		178	1.94	35	29	6			
		179	2.05	32	28	4			
		180	2.33	17	26	-9			
		181	2.40	26	31	-5			
	H						1.69	1.78	0.09
		182	1.23	19	10	9			
		183	1.61	18	22	-4			
		184	2.05	34	32	2			
		185	2.43	39	42	-3			
		186	3.42	46	48	-2			
	I						2.02	2.10	0.08
		187	1.58	17	9	8			
		188	2.14	17	21	-4			
		189	2.35	43	34	9			
		190	2.35	31	41	-10			
		191	2.61	39	39	0			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		192	2.94	40	46	-6			
Black	A						1.21	1.22	0.01
		193	0.74	13	8	5			
		194	0.88	20	14	6			
		195	1.35	17	25	-8			
		196	1.51	37	35	2			
		197	1.73	42	47	-5			
	B						0.78	0.78	0.00
		198	0.47	7	2	5			
		199	0.63	16	13	3			
		200	0.70	25	25	0			
		201	0.97	39	37	2			
		202	1.15	39	46	-7			
	C						1.30	1.28	-0.02
		203	0.85	11	9	2			
		204	1.08	24	19	5			
		205	1.26	25	28	-3			
		206	1.46	30	27	3			
		207	1.53	26	34	-8			
		208	2.01	43	46	-3			
	F						1.08	1.10	0.02
		209	0.44	10	5	5			
		210	0.87	15	9	6			
		211	1.09	20	25	-5			
		212	1.30	43	37	6			
		213	1.73	38	46	-8			
	G						1.06	1.06	0.00
		214	0.65	8	5	3			
		215	0.89	23	20	3			
		216	1.20	27	29	-2			
		217	1.23	34	34	0			
		218	1.45	39	43	-4			
	H						1.02	1.05	0.03
		219	0.69	10	3	7			
		220	0.78	23	11	12			
		221	0.84	16	17	-1			
		222	1.10	25	28	-3			
		223	1.25	41	37	4			
		224	1.58	38	46	-8			
	I						0.94	0.94	0.00
		225	0.65	8	6	2			
		226	0.77	22	18	4			
		227	1.02	33	33	0			
		228	1.23	33	38	-5			
		229	1.39	42	46	-4			
Moderate reddish brown	A						0.99	1.01	0.02

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		230	0.61	19	7	12			
		231	0.80	13	22	-9			
		232	1.17	38	31	7			
		233	1.31	38	37	1			
		234	1.56	26	42	-16			
	B						1.15	1.16	0.01
		235	0.78	18	7	11			
		236	0.97	12	18	-6			
		237	1.10	24	22	2			
		238	1.36	33	34	-1			
		239	1.50	35	43	-8			
		240	2.11	48	48	0			
	C						1.57	1.51	-0.06
		241	1.01	7	4	3			
		242	1.25	11	13	-2			
		243	1.45	24	24	0			
		244	1.78	36	35	1			
		245	1.95	35	45	-10			
	F						1.06	1.11	0.05
		246	0.90	23	17	6			
		247	1.01	28	29	-1			
		248	1.22	24	25	-1			
		249	1.34	19	31	-12			
		250	1.57	46	35	11			
		251	1.95	35	49	-14			
	G						0.95	0.95	0.00
		252	0.62	8	2	6			
		253	0.74	20	12	8			
		254	0.93	14	24	-10			
		255	1.10	42	37	5			
		256	1.27	38	46	-8			
		257	1.54	47	48	-1			
	H						1.38	1.35	-0.03
		258	0.62	5	4	1			
		259	1.01	23	10	13			
		260	1.29	13	23	-10			
		261	1.54	28	29	-1			
		262	1.83	41	46	-5			
	I						0.91	0.93	0.02
		263	0.60	11	6	5			
		264	0.84	20	15	5			
		265	0.93	25	27	-2			
		266	1.12	39	39	0			
		267	1.32	41	46	-5			
Dark bluish green	A						0.97	1.07	0.10
		268	0.78	15	9	6			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		269	1.03	32	24	8			
		270	1.33	29	39	-10			
		271	1.35	44	39	5			
		272	1.63	38	47	-9			
	B						3.55	3.63	0.08
		273	2.75	14	9	5			
		274	3.58	23	26	-3			
		275	4.07	40	32	8			
		276	4.21	31	39	-8			
		277	5.43	38	42	-4			
	C						1.40	1.43	0.03
		278	0.86	11	5	6			
		279	1.15	21	15	6			
		280	1.37	18	23	-5			
		281	1.62	33	29	4			
		282	1.69	34	36	-2			
		283	1.96	39	46	-7			
	F						2.20	2.24	0.04
		284	1.37	12	4	8			
		285	1.70	14	14	0			
		286	2.42	23	24	-1			
		287	2.77	41	43	-2			
		288	3.71	47	49	-2			
	G						1.35	1.37	0.02
		289	0.66	4	1	3			
		290	0.97	11	6	5			
		291	1.10	11	11	0			
		292	1.32	18	18	0			
		293	1.43	26	28	-2			
		294	1.77	47	47	0			
	H						1.43	1.51	0.08
		295	1.08	13	4	9			
		296	1.23	13	18	-5			
		297	1.44	42	36	6			
		298	1.68	36	34	2			
		299	2.15	26	34	-8			
		300	2.24	39	42	-3			
	I						1.19	1.18	-0.01
		301	0.65	5	2	3			
		302	1.10	30	29	1			
		303	1.44	41	38	3			
		304	1.75	34	42	-8			
		305	2.19	39	47	-8			
Very dark red	A						1.01	1.06	0.05
		306	0.75	12	9	3			
		307	0.98	38	28	10			
		308	1.12	28	29	-1			
		309	1.29	23	32	-9			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		310	1.53	36	39	-3			
	B						1.69	1.73	0.04
		311	1.19	14	7	7			
		312	1.54	14	12	2			
		313	1.88	29	31	-2			
		314	2.18	43	41	2			
		315	2.29	42	50	-8			
	C						1.71	1.63	-0.08
		316	0.72	4	1	3			
		317	1.26	12	11	1			
		318	1.57	21	21	0			
		319	1.72	32	29	3			
		320	1.98	28	39	-11			
	F						1.45	1.42	-0.03
		321	0.67	5	1	4			
		322	1.09	12	7	5			
		323	1.09	16	10	6			
		324	1.47	14	21	-7			
		325	1.62	40	42	-2			
	G						1.65	1.65	0.00
		326	0.98	7	4	3			
		327	1.28	20	20	0			
		328	1.46	29	27	2			
		329	1.75	22	24	-2			
		330	1.96	26	28	-2			
		331	2.28	42	43	-1			
	H						1.80	1.80	0.00
		332	1.33	13	11	2			
		333	1.62	18	16	2			
		334	1.74	24	24	0			
		335	2.05	32	28	4			
		336	2.14	32	39	-7			
		337	2.32	42	45	-3			
	I						1.30	1.33	0.03
		338	0.76	8	3	5			
		339	0.98	12	13	-1			
		340	1.29	33	26	7			
		341	1.56	29	35	-6			
		342	1.90	43	43	0			
Dark blue	A						1.01	1.02	0.01
		343	0.70	12	7	5			
		344	0.94	24	26	-2			
		345	1.04	29	30	-1			
		346	1.33	34	36	-2			
		347	1.47	42	41	1			
		348	1.59	41	47	-6			
	B						1.40	1.40	0.00

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		349	1.15	13	11	2			
		350	1.31	26	27	-1			
		351	1.78	39	40	-1			
		352	2.38	45	47	-2			
		353	2.69	46	48	-2			
	C						1.18	1.15	-0.03
		354	0.48	1	1	0			
		355	0.82	11	6	5			
		356	1.11	24	27	-3			
		357	1.52	43	43	0			
		358	2.09	46	49	-3			
	F						0.93	0.98	0.05
		359	0.77	16	13	3			
		360	1.00	25	24	1			
		361	1.11	35	37	-2			
		362	1.28	45	37	8			
		363	1.51	33	43	-10			
		364	1.77	46	46	0			
	G						1.69	1.73	0.04
		365	1.07	7	4	3			
		366	1.40	26	18	8			
		367	1.75	21	24	-3			
		368	1.91	27	30	-3			
		369	2.29	43	43	0			
	H						1.04	1.05	0.01
		370	0.51	8	3	5			
		371	0.92	11	10	1			
		372	0.98	25	22	3			
		373	1.08	39	32	7			
		374	1.29	30	40	-10			
		375	1.44	37	42	-5			
	I						1.37	1.74	0.37
		376	1.45	26	13	13			
		377	1.68	24	31	-7			
		378	1.73	33	24	9			
		379	1.97	19	33	-14			
		380	2.19	38	31	7			
		381	2.51	30	44	-14			
Moderate blue	F						1.01	1.06	0.05
		382	0.96	20	14	6			
		383	1.07	28	30	-2			
		384	1.31	42	41	1			
		385	1.60	42	46	-4			
		386	1.67	46	49	-3			
	G						1.13	1.18	0.05
		387	0.91	29	19	10			
		388	1.01	26	22	4			
		389	1.19	10	17	-7			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Unfiltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfiltred and Filtered	Unfiltred T50	Filtered T50	T50 diff btwn Unfiltred and Filtered
		390	1.29	26	25	1			
		391	1.53	38	42	-4			
	H						0.87	0.87	0.00
		392	0.59	6	6	0			
		393	0.80	30	22	8			
		394	0.94	31	35	-4			
		395	1.12	34	39	-5			
		396	1.57	46	48	-2			
	I						1.53	1.53	0.00
		397	1.12	6	2	4			
		398	1.48	9	13	-4			
		399	1.52	50	38	12			
		400	1.84	35	45	-10			
		401	2.03	36	44	-8			
		402	2.51	48	49	-1			
Moderate greenish blue	F						1.18	1.24	0.06
		403	1.00	21	8	13			
		404	1.11	22	19	3			
		405	1.31	22	31	-9			
		406	1.57	46	44	2			
		407	1.79	44	48	-4			
	G						1.00	1.00	0.00
		408	0.68	6	3	3			
		409	0.90	22	21	1			
		410	1.08	35	34	1			
		411	1.31	34	41	-7			
		412	1.40	45	47	-2			
	H						1.20	1.19	-0.01
		413	0.81	9	4	5			
		414	1.09	16	15	1			
		415	1.20	32	32	0			
		416	1.37	42	36	6			
		417	1.60	29	45	-16			
	I						0.86	0.87	0.01
		418	0.72	14	11	3			
		419	0.93	29	33	-4			
		420	1.04	46	42	4			
		421	1.17	37	42	-5			
		422	1.33	32	43	-11			
		423	0.06	2	0	2			
Moderate bluish green	F						1.28	1.28	0.00
		424	0.96	14	10	4			
		425	1.24	23	23	0			
		426	1.43	32	32	0			
		427	1.51	29	36	-7			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		428	1.79	47	48	-1			
	G						1.47	1.49	0.02
		429	0.92	6	1	5			
		430	1.20	3	5	-2			
		431	1.54	33	22	11			
		432	1.59	29	40	-11			
		433	1.77	46	45	1			
		434	2.14	46	50	-4			
	H						1.29	1.29	0.00
		435	0.84	16	8	8			
		436	1.04	15	13	2			
		437	1.16	23	21	2			
		438	1.36	20	27	-7			
		439	1.62	32	36	-4			
		440	1.77	45	46	-1			
	I						1.15	1.17	0.02
		441	0.92	13	10	3			
		442	1.16	28	25	3			
		443	1.49	45	42	3			
		444	1.42	32	38	-6			
		445	1.54	44	48	-4			
Medium gray	F						1.01	1.03	0.02
		446	0.46	4	2	2			
		447	0.88	21	15	6			
		448	1.07	21	23	-2			
		449	1.20	29	33	-4			
		450	1.29	43	43	0			
		451	1.37	46	48	-2			
	G						1.02	1.01	-0.01
		452	0.60	2	1	1			
		453	0.75	12	11	1			
		454	1.30	42	46	-4			
		455	1.23	37	34	3			
		456	1.44	47	49	-2			
		457	1.68	47	50	-3			
	H						0.87	0.87	0.00
		458	0.59	5	3	2			
		459	0.78	14	11	3			
		460	0.88	25	29	-4			
		461	1.02	44	44	0			
		462	1.22	45	47	-2			
	I						0.94	0.93	-0.01
		463	0.57	5	2	3			
		464	0.80	10	10	0			
		465	0.93	36	30	6			
		466	1.04	32	37	-5			
		467	1.27	38	45	-7			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
Light brown	F						1.45	1.49	0.04
		468	1.27	14	12	2			
		469	1.42	29	23	6			
		470	1.69	31	35	-4			
		471	1.82	37	41	-4			
		472	1.97	45	45	0			
		473	2.34	45	50	-5			
	G						1.08	1.07	-0.01
		474	0.70	6	3	3			
		475	0.87	10	7	3			
		476	0.95	18	18	0			
		477	1.20	38	40	-2			
		478	1.63	45	48	-3			
	H						1.17	1.15	-0.02
		479	0.67	4	3	1			
		480	0.88	14	12	2			
		481	1.20	34	28	6			
		482	1.36	30	38	-8			
		483	1.56	38	43	-5			
	I						1.02	1.01	-0.01
		484	0.62	8	2	6			
		485	0.80	12	12	0			
		486	0.94	25	23	2			
		487	1.20	38	37	1			
		488	1.42	38	46	-8			
Grayish purple	F						1.06	1.02	-0.04
		489	0.69	15	3	12			
		490	0.76	7	12	-5			
		491	0.92	16	15	1			
		492	1.01	26	25	1			
		493	1.19	31	38	-7			
	G						1.40	1.42	0.02
		494	1.08	16	5	11			
		495	1.19	18	15	3			
		496	1.31	24	20	4			
		497	1.51	21	27	-6			
		498	1.77	41	44	-3			
	H						1.02	1.03	0.01
		499	0.84	14	10	4			
		500	0.99	29	23	6			
		501	1.04	29	31	-2			
		502	1.19	28	33	-5			
		503	1.27	35	39	-4			
	I						1.38	1.25	-0.13
		504	0.64	13	3	10			
		505	0.96	14	14	0			
		506	1.08	23	23	0			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		507	1.36	27	31	-4			
		508	1.38	32	30	2			
		509	1.58	21	35	-14			
Dark reddish orange	F						1.65	1.61	-0.04
		510	1.30	8	4	4			
		511	1.44	18	14	4			
		512	1.55	26	28	-2			
		513	1.66	33	37	-4			
		514	1.92	43	37	6			
		515	2.25	27	46	-19			
	G						0.39	0.90	0.51
		516	1.03	26	24	2			
		517	1.19	16	22	-6			
		518	0.09	28	19	9			
		519	1.48	30	30	0			
		520	1.67	38	41	-3			
	H						1.57	1.59	0.02
		521	1.29	22	5	17			
		522	1.43	8	10	-2			
		523	1.53	15	15	0			
		524	1.59	32	33	-1			
		525	1.90	42	45	-3			
	I						1.18	1.25	0.07
		526	1.15	28	9	19			
		527	1.19	21	31	-10			
		528	1.41	33	36	-3			
		529	1.53	36	43	-7			
		530	1.62	45	44	1			
		531	1.91	45	49	-4			
Moderate yellow	F						1.45	1.47	0.02
		532	1.12	14	9	5			
		533	1.56	27	27	0			
		534	1.69	36	36	0			
		535	1.88	37	47	-10			
		536	2.13	46	50	-4			
	G						1.39	1.40	0.01
		537	1.00	12	9	3			
		538	1.27	23	20	3			
		539	1.49	30	25	5			
		540	1.59	28	36	-8			
		541	1.87	37	42	-5			
		542	1.93	43	47	-4			
	H						1.27	1.30	0.03
		543	0.90	8	5	3			
		544	1.20	30	16	14			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		545	1.29	21	28	-7			
		546	1.44	31	35	-4			
		547	1.81	44	46	-2			
	I						1.16	1.15	-0.01
		548	0.79	4	5	-1			
		549	0.99	21	16	5			
		550	1.27	38	28	10			
		551	1.37	27	35	-8			
		552	1.59	44	46	-2			
Grayish yellow green	F						0.80	0.96	0.16
		553	0.94	7	17	-10			
		554	0.04	21	5	16			
		555	1.20	36	33	3			
		556	1.48	45	42	3			
		557	1.66	33	48	-15			
	G						1.32	1.43	0.11
		558	1.23	20	10	10			
		559	1.39	23	26	-3			
		560	1.49	35	35	0			
		561	1.82	37	39	-2			
		562	2.25	38	45	-7			
	H						1.06	1.09	0.03
		563	0.78	19	2	17			
		564	0.89	9	17	-8			
		565	1.08	29	24	5			
		566	1.27	34	36	-2			
		567	1.50	36	46	-10			
		568	1.61	49	50	-1			
	I						1.00	1.01	0.01
		569	0.65	5	2	3			
		570	0.80	22	12	10			
		571	1.03	16	26	-10			
		572	1.27	43	39	4			
		573	1.37	42	48	-6			
		574	1.51	48	50	-2			
Moderate blue	D						0.99	1.01	0.02
		575	0.74	13	12	1			
		576	1.13	31	27	4			
		577	1.22	35	40	-5			
		578	1.50	46	46	0			
		579	1.93	47	50	-3			
	E						3.31	3.22	-0.09
		580	2.11	4	1	3			
		581	2.60	12	8	4			
		582	2.82	27	18	9			
		583	3.01	18	25	-7			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		584	3.72	29	34	-5			
Moderate greenish blue	A						1.01	1.01	0.00
		585	0.59	4	1	3			
		586	0.69	25	10	15			
		587	0.76	14	22	-8			
		588	0.90	21	25	-4			
		589	1.28	33	35	-2			
	D						1.55	1.53	-0.02
		590	1.02	4	4	0			
		591	1.35	25	19	6			
		592	1.55	27	30	-3			
		593	1.90	33	36	-3			
		594	2.11	42	46	-4			
Medium gray	B						0.87	0.87	0.00
		595	0.52	7	4	3			
		596	0.76	18	16	2			
		597	0.91	27	26	1			
		598	0.94	31	33	-2			
		599	1.21	41	47	-6			
Light brown	C						1.61	1.62	0.01
		600	0.93	4	2	2			
		601	1.37	20	11	9			
		602	1.55	21	24	-3			
		603	1.85	32	34	-2			
		604	1.95	38	41	-3			
Grayish purple	C						1.78	1.67	-0.11
		605	1.01	2	0	2			
		606	1.22	8	7	1			
		607	1.23	17	11	6			
		608	1.44	16	17	-1			
		609	1.65	22	22	0			
		610	1.88	30	34	-4			
Dark reddish orange	C						1.75	1.73	-0.02
		611	1.04	5	1	4			
		612	1.35	12	1	11			
		613	1.42	17	9	8			
		614	1.40	0	6	-6			
		615	1.77	24	27	-3			
		616	1.93	36	39	-3			
Moderate bluish green	A						1.03	0.98	-0.05
		617	0.29	6	1	5			
		618	0.83	14	11	3			
		619	0.84	17	19	-2			
		620	0.97	26	26	0			

Table XIX (Cont.)

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Unfltred Freq. of Rejectn.	Filtered Freq. of Rejectn.	Freq. diff btwn Unfltred and Filtered	Unfltred T50	Filtered T50	T50 diff btwn Unfltred and Filtered
		621	1.18	31	37	-6			
	B						2.58	2.49	-0.09
		622	1.60	8	3	5			
		623	2.03	14	14	0			
		624	2.21	17	16	1			
		625	2.43	23	22	1			
		626	2.94	32	38	-6			
	C						1.22	1.22	0.00
		627	0.81	8	5	3			
		628	1.03	19	18	1			
		629	1.23	25	26	-1			
		630	1.56	40	38	2			
		631	1.73	41	47	-6			
	D						1.78	1.78	0.00
		632	1.12	10	7	3			
		633	1.52	23	12	11			
		634	1.69	16	28	-12			
		635	2.20	36	43	-7			
		636	2.19	36	31	5			
	E						1.81	1.83	0.02
		637	1.17	9	7	2			
		638	1.59	25	17	8			
		639	1.78	23	23	0			
		640	1.94	26	27	-1			
		641	2.11	31	35	-4			
		642	2.50	41	45	-4			
Statistics									
Number of Cases						642	119	119	119
Maximum						19	5.1333	4.46	1.274
Minimum						-19	0.3356	0.78	-0.673
Mean						-0.131	1.4176	1.440	0.022
Stdev						5.767	0.5855	0.534	0.150

V. Filtering Effectiveness and Data Pooling

A. Filtering Effectiveness

There were three independent goals for using a filter on the response data: 1) to not destroy, bias or otherwise interfere with the underlying information in the data; 2) to remove the effect which the intra-observer noise had on the calculation of the inter-observer model fit; 3) to remove the effect which the intra-observer noise had in the calculation of the inter-observer variance. It was determined, as described in Chapter III, that the major differences between Phase I and Phase II was the observer task difficulty and that this was causing an increase in intra-observer noise. The Probit analysis procedure was not designed to handle such noise and was misinterpreting it as between-observer variation. By carefully preserving the data's integrity while attenuating observer uncertainty, derived metrics should be more robust, have had more validity and, if the statistical differences between the Phase I and II populations became insignificant, the filtering would allow a pooling of the results.

The Phase I data were considered relatively free from intra-observer noise, while the Phase II data were, in contrast, very noisy. Since Phase I had little intra-observer noise, a filter that attenuated such noise should have had little impact on its calculated T50's, χ^2 probability terms and standard deviations. Based on the experience of Stroka's[20] experiments, noise like that found in the Phase II data would not have destroyed the accuracy of the pre-filtered T50's. The first goal outlined above was to preserve the underlying information. This was considered satisfied once it was shown that the filtering procedure had little impact on Phase I and Phase II T50's, Phase I χ^2 probability terms and Phase I standard deviations. Every Phase I T50 and 91% of the Phase II T50's changed by less than 0.10 ΔE^*_{ab} units. Only three Phase II T50's changed by more than 0.20 ΔE^*_{ab} units. For the Phase I data, there was little effective change between unfiltered and filtered χ^2 probability terms: there was an average drop of 0.04 units and a net decrease of 1 vector passing the good model fit test of 0.05. Standard deviations for Phase I vectors also changed little with an average decrease of 0.05 units after filtering. The filter was, thus, considered to have satisfied the first goal.

The second goal was to remove the effect of noisy observer responses from the calculation of the goodness of model fit as manifested in the χ^2 probability term. As was hoped, the filtering procedure had a large impact on the Phase II χ^2 probability terms with an average increase of 0.12 units. This resulted in 54% more vectors passing the goodness of fit measured by comparing χ^2 probability terms to 0.05. The second goal was satisfied.

The standard deviation value was a measurement of the precision with which the population performed the requested task. The third goal for applying the filter was to decrease the magnitude of the standard deviations by removing within-observer noise from the calculation. After filtering, Phase II standard deviations fell by an average of 0.35 units, thus satisfying the third goal.

B. Data Pooling

Having been satisfied that the three goals for appropriate filter application had been met, t-tests were performed on the filtered data in a similar fashion to those summarized for unfiltered data in Table XII. Metrics associated with the filtered Moderate Bluish Green color center were compared to determine if the Phase I and II populations were sufficiently similar allowing a pooling of the results. Table XX shows the results of these t-tests.

Table XX: T-test Comparison of Phase I and II Filtered Moderate Bluish Green Responses

Moderate Bluish Green was the only color center fully repeated between the Phases.

n = 5

	T50			χ^2 prob.			stdev		
	mean	stdev	prob	mean	stdev	prob	mean	stdev	prob
Phase I	1.60	0.50		0.78	0.13		0.32	0.06	
			0.86			0.20			0.06
Phase II	1.66	1.59		0.51	0.38		0.50	0.14	

At a 95% confidence level, probabilities of greater than 0.05 imply that the two populations were the same. With respect to T50, χ^2 probability term and standard deviation, the filtered Phase I and Phase II populations were not significantly different. Thus, a pooling of the results was enabled.

Table XXI shows a summary of the pooled Phase I and Phase II filtered data sets.

Table XXI: Filtering results summary:

Corrected version of first eight columns of previously published summary table (Berns, et al[4], Table IV). Column three denotes which experiment generated the raw data. When both experiments probed the same vector, that with highest precision was chosen.

Color Center			T_{50}	LFL	UFL	S	P:CHISQ
Moderate blue	A	I	0.96	0.88	1.04	0.43	0.04
	B	I	1.37	1.28	1.44	0.40	0.01
	C	I	1.55	1.45	1.61	0.38	0.69
	D	II	1.01	0.93	1.08	0.34	0.30
	E	II	3.22	3.08	3.39	0.73	0.06
	F	II	1.06	0.98	1.13	0.32	0.14
	G	II	1.18			0.56	0.01
	H	II	0.87	0.80	0.92	0.32	0.08
	I	II	1.53	0.57	1.94	0.34	0.00
Moderate greenish blue	A	I	0.78	0.73	0.85	0.31	0.53
	B	I	1.62	1.53	1.68	0.37	0.65
	C	I	1.62	1.24	2.00	0.38	0.00
	D	II	1.53	1.45	1.61	0.44	0.20
	E	I	1.48	1.39	1.60	0.49	0.00
	F	II	1.24	1.18	1.29	0.29	0.67
	G	II	1.00	0.95	1.05	0.27	0.18
	H	II	1.19	1.14	1.24	0.29	0.20
	I	II	0.87	0.60	1.00	0.30	0.02
Medium gray	A	I	0.94	0.88	0.99	0.21	1.00
	B	II	0.87	0.82	0.91	0.23	0.79
	C	I	1.32	1.23	1.42	0.41	0.45
	D	I	0.93	0.87	0.97	0.21	0.85
	E	I	1.27	1.22	1.37	0.28	0.93
	F	II	1.03	0.98	1.08	0.27	0.12
	G	II	1.01	0.87	1.13	0.25	0.04
	H	II	0.87	0.83	0.90	0.18	0.07
	I	II	0.93	0.89	0.97	0.22	0.14
Moderate bluish green	A	I	0.96	0.89	1.00	0.25	0.92
	A	II	0.98	0.92	1.05	0.31	0.37
	B	I	2.28	2.21	2.37	0.41	0.87
	B	II	2.49	2.38	2.63	0.63	0.80
	C	I	1.30	1.25	1.38	0.35	0.84
	C	II	1.22	1.15	1.29	0.37	0.41
	D	I	1.68	1.61	1.73	0.29	0.66
	D	II	1.78	1.15	2.48	0.60	0.01
	E	I	1.77	1.71	1.84	0.31	0.62
	E	II	1.83	1.73	1.92	0.57	0.96
	F	II	1.28	1.21	1.34	0.35	0.74
	G	II	1.49	1.36	1.59	0.23	0.04
	H	II	1.29	1.22	1.36	0.43	0.59
	I	II	1.17	1.10	1.22	0.29	0.38
Light brown	A	I	0.90	0.75	1.11	0.25	0.71
	B	I	1.39	1.32	1.46	0.28	0.97

Table XXI (Cont.)

Color Center			T_{50}	LFL	UFL	S	P:CHISQ
	C	II	1.62	1.55	1.70	0.39	0.68
	D	I	1.59	1.51	1.68	0.42	0.25
	E	I	1.14	1.09	1.18	0.18	0.94
	F	II	1.49	1.42	1.56	0.35	0.90
	G	II	1.07	0.93	1.27	0.25	0.04
	H	II	1.15	1.08	1.21	0.34	0.81
	I	II	1.01	0.96	1.06	0.27	0.56
Grayish purple	A	I	0.94	0.87	1.03	0.28	0.89
	B	I	1.47	1.41	1.53	0.31	0.31
	C	II	1.67	1.59	1.80	0.49	0.83
	D	I	1.23	1.17	1.29	0.31	0.34
	E	I	2.88	2.77	2.98	0.53	0.60
	F	II	1.02	0.97	1.07	0.26	0.25
	G	II	1.42	1.36	1.48	0.32	0.34
	H	II	1.03	0.98	1.08	0.29	0.28
	I	II	1.25	1.17	1.33	0.49	0.41
Dark reddish orange	A	I	0.94	0.81	1.09	0.31	0.61
	B	I	1.94	1.88	2.01	0.30	0.27
	C	II	1.73	1.68	1.79	0.28	0.26
	D	I	2.01	1.92	2.11	0.59	0.27
	E	I	1.32	1.24	1.39	0.36	0.00
	F	II	1.61	1.34	1.83	0.37	0.00
	G	II	0.90			1.71	0.02
	H	II	1.59	1.55	1.63	0.22	0.06
	I	II	1.25	0.86	1.39	0.29	0.00
Moderate yellow	A	I	1.19	1.12	1.25	0.32	0.70
	B	I	1.44	1.38	1.52	0.32	0.80
	C	I	2.20	2.13	2.30	0.48	0.90
	D	I	1.63	1.56	1.70	0.29	0.94
	E	I	1.28	1.24	1.35	0.23	0.93
	F	II	1.47	1.39	1.53	0.31	0.24
	G	II	1.40	1.32	1.46	0.41	0.48
	H	II	1.30	1.24	1.36	0.33	0.47
	I	II	1.15	1.10	1.20	0.28	0.32
Grayish yellow green	A	I	0.86	0.64	1.07	0.33	0.00
	B	I	1.17	1.09	1.21	0.27	0.07
	C	I	1.44	1.37	1.50	0.32	0.19
	D	I	1.21	1.15	1.26	0.25	0.99
	E	I	1.72	1.61	1.77	0.31	0.44
	F	II	0.96	0.43	1.26	0.56	0.03
	G	II	1.43	-1.06	1.80	0.53	0.01
	H	II	1.09	1.04	1.14	0.26	0.06
	I	II	1.01	0.97	1.06	0.24	0.23
Black	A	II	1.22	1.13	1.30	0.47	0.11
	B	II	0.78	0.74	0.83	0.25	0.10
	C	II	1.28	1.20	1.37	0.54	0.50
	F	II	1.10	1.02	1.18	0.43	0.14

Table XXI (Cont.)

Color Center			T_{50}	LFL	UFL	S	P:CHISQ
	G	II	1.06	1.00	1.13	0.37	0.42
	H	II	1.05	1.00	1.11	0.33	0.31
	I	II	0.94	0.88	1.00	0.32	0.25
Light bluish green	A	II	1.26			0.99	0.00
	B	II	2.38	2.26	2.50	0.63	0.43
	C	II	1.78	1.71	1.86	0.40	0.51
	F	II	1.31	1.00	1.58	0.39	0.00
	G	II	1.68	1.45	1.91	0.29	0.02
	H	II	1.67	1.58	1.75	0.45	0.12
	I	II	1.47	1.40	1.53	0.31	0.15
Moderate reddish brown	A	II	1.01	0.91	1.09	0.50	0.24
	B	II	1.16	1.08	1.23	0.43	0.25
	C	II	1.51	1.44	1.58	0.38	0.61
	F	II	1.11	0.63	1.32	0.60	0.04
	G	II	0.95	0.91	1.00	0.26	0.15
	H	II	1.35	1.27	1.43	0.45	0.24
	I	II	0.93	0.88	0.98	0.27	0.68
Dark bluish green	A	II	1.07	1.00	1.14	0.35	0.95
	B	II	3.63	3.35	3.86	1.37	0.06
	C	II	1.43	1.35	1.50	0.45	0.62
	F	II	2.24	2.11	2.38	0.69	0.09
	G	II	1.37	1.31	1.43	0.32	0.59
	H	II	1.51			0.74	0.00
	I	II	1.18	0.47	1.55	0.51	0.00
Brilliant greenish blue	A	II	1.23	0.55	2.27	0.80	0.00
	B	II	1.67	1.54	1.77	0.60	0.10
	C	II	2.07	1.95	2.18	0.62	0.11
	F	II	1.00	0.17	1.40	0.30	0.00
	G	II	1.67	1.26	1.96	0.51	0.04
	H	II	1.36	1.13	1.52	0.36	0.02
	I	II	2.19	2.11	2.27	0.46	0.47
Very dark red	A	II	1.06	0.96	1.15	0.52	0.10
	B	II	1.73	1.45	1.96	0.39	0.03
	C	II	1.63	1.55	1.72	0.45	0.98
	F	II	1.42			0.33	0.02
	G	II	1.65	1.25	2.09	0.72	0.01
	H	II	1.80	1.71	1.88	0.52	0.18
	I	II	1.33	1.25	1.41	0.46	0.44
Moderate purplish pink	A	II	1.34	1.27	1.41	0.43	0.06
	B	II	2.41	2.32	2.52	0.63	0.29
	C	II	1.91	1.83	2.00	0.48	0.15
	F	II	1.68	1.34	2.69	0.53	0.02
	G	II	1.58	1.47	1.73	0.55	0.06
	H	II	1.58	1.51	1.65	0.38	0.77
	I	II	1.69	1.60	1.79	0.50	0.18
Dark blue	A	II	1.02	0.94	1.08	0.42	0.14
	B	II	1.40	0.81	1.72	0.62	0.05

Table XXI (Cont.)

Color Center			T_{50}	LFL	UFL	S	P:CHISQ
	C	II	1.15	1.08	1.23	0.35	0.11
	F	II	0.98	0.88	1.06	0.48	0.19
	G	II	1.73	1.63	1.83	0.55	0.40
	H	II	1.05	1.00	1.11	0.33	0.08
	I	II	1.74	1.09	2.00	0.74	0.04
Light gray	A	II	1.30	1.13	1.41	0.66	0.36
	B	II	1.02	0.88	1.12	0.50	0.28
	C	II	1.83	1.71	1.93	0.57	0.30
	F	II	1.26	1.06	1.44	0.43	0.03
	G	II	1.36	1.27	1.44	0.50	0.85
	H	II	1.01	0.82	1.22	0.26	0.02
	I	II	1.55	1.46	1.64	0.52	0.60
Strong orange yellow	A	II	1.61	1.44	1.73	0.74	0.70
	B	II	1.88	1.36	2.22	0.63	0.04
	C	II	4.46	4.13	5.30	0.98	0.17
	F	II	2.10	1.92	2.25	0.98	0.55
	G	II	2.06			0.88	0.00
	H	II	1.78	1.62	1.92	0.79	0.48
	I	II	2.10	1.99	2.19	0.56	0.13

Statistics					
Number of Cases	156	150	150	156	156
Minimum	0.780	-1.060	0.830	0.180	0.000
Maximum	4.460	4.130	5.300	1.710	1.000
Mean	1.437	1.281	1.569	0.433	0.339
Standard Deviation	0.513	0.558	0.583	0.208	0.315

VI. Color-Difference Equation Testing

A. Comparison of Filtered Data With Color-Difference Formulae

In order to compare the visual scaling data with various color-difference formulae, each formula was used to calculate the color-distance between the XYZ tristimulus values for each color center and the XYZ tristimulus values which were exactly T50 distance in the positive direction of each vector from the color center. XYZ[8] Euclidean distance, CIELAB, CIELUV[7], SVF[9], FMC2[10,11], BFD(1:1)[12,13], CMC(1:1)[14,15], and the NBS Unit of Color Difference[16,17] were each tested. 'C' subroutines which accepted a pair of XYZ values and returned the magnitude of the respective metric were implemented for all the above color-space formulae. Source code for these routines are included in Appendix C.

The results of calculating the eight color-difference formulae between the color centers and the T50 points appear in Table A-III. Recall that each T50 distance represents a single industrial-sized color-difference visual unit. If any of the color-difference formulae had returned an identical number for every row in Table A-III, then it would have been easy to declare that equation as being uniform across color-space with respect to human industrial-sized color-difference perception.

In order to normalize the color-difference values for further comparison, the mean for each formula was divided out from each calculated color-difference. From that the minimum, maximum, range, mean and standard deviation were calculated. See Table XXII.

Table XXII: Statistics on Normalized Color-Difference Calculations

	dXYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
Number of Cases	156	156	156	156	156	156	156	156
Minimum	0.07	0.5	0.392	0.354	0.537	0.331	0.576	0.066
Maximum	4.478	2.697	2.489	2.558	3.111	2.279	2.125	2.538
Range	4.408	2.197	2.098	2.204	2.575	1.948	1.549	2.472
Mean	1	1	1	1	1	1	1	1
Standard Deviation	0.778	0.346	0.389	0.396	0.358	0.388	0.286	0.476

A metric which reflected perfect correlation with human industrial-sized color-difference perception, as measured by these studies, would have minimum and maximum of one, range of zero, and standard deviation of zero. CMC(1:1) has both the minimum and maximum closest to one, the lowest range, and the lowest standard deviation. Thus, for every category explicit in Table XXII, CMC(1:1) performs the best. There is no clear second best: SVF has the second lowest standard deviation, FMC2 has the second lowest range, CIELAB has the minimum which is second closest to one and FMC2 has the maximum which is second closest to 1.

The Kolmogrov-Smirnov test[66] allows for the comparison of independent distributions to determine whether they came from the same population. The normalized color-difference metrics treated above were compared to each other using the Kolmogrov-Smirnov test in order to attach a significance to the differences displayed in Table XXII. To a confidence level of 95%, the hypothesis that each pair of distributions emerged from the same population was tested. The performance of CMC(1:1) was found to be statistically distinguishable from all of the other metrics.

Table XXIII: Kolmogorov-Smirnov Test

n = 158

Ho: samples came from identical continuous distributions

equal = accept Ho; reject = reject Ho

	XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
XYZ		reject	reject	reject	reject	reject	reject	reject
SVF	reject		reject	reject	equal	equal	reject	reject
NBS	reject	reject		equal	equal	equal	reject	reject
L*u*v*	reject	reject	equal		equal	equal	reject	reject
L*a*b*	reject	equal	equal	equal		equal	reject	reject
FMC2	reject	equal	equal	equal	equal		reject	equal
CMC(1:1)	reject	reject	reject	reject	reject	reject		reject
BFD(1:1)	reject	reject	reject	reject	reject	equal	reject	

Another way to visualize the color-difference metric comparisons is through a cumulative histogram. The normalized color-differences associated with each formula were sorted in ascending order and a cumulative percentage assigned to each normalized measurement. A perfect match to industrial-sized color-difference perception would display a step function which transitions at one.

A study of Figure 9 reveals that unlike the other formulae, dXYZ does not appear to have any trends connected with unity. BFD, and to a lesser extent, FMC2, appear to have bimodal histograms indicating that these formulae are each reacting in two distinct ways to color-difference of industrial-sized magnitude.

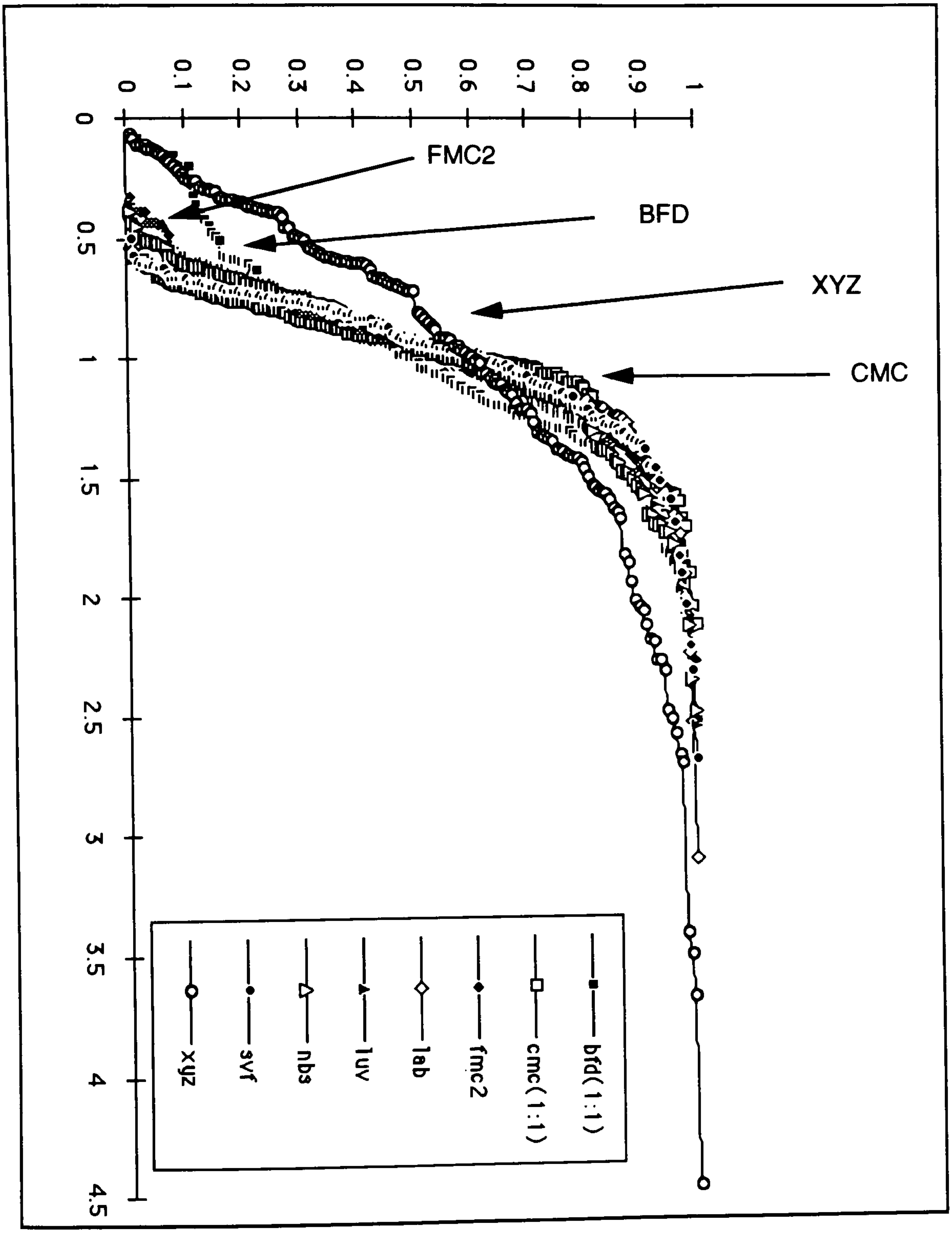

Figure 9: Combined Color-difference Cumulative Histogram

It is interesting to note that by the criteria explained above, CMC(1:1) has outperformed BFD(1:1). As introduced in Chapter II, Section B, BFD(l:c) was specifically designed as an improvement to CMC(l:c). The above results imply that for industrial sized color-differences, the changes were not successful in improving the correspondence between human perception and calculated metric.

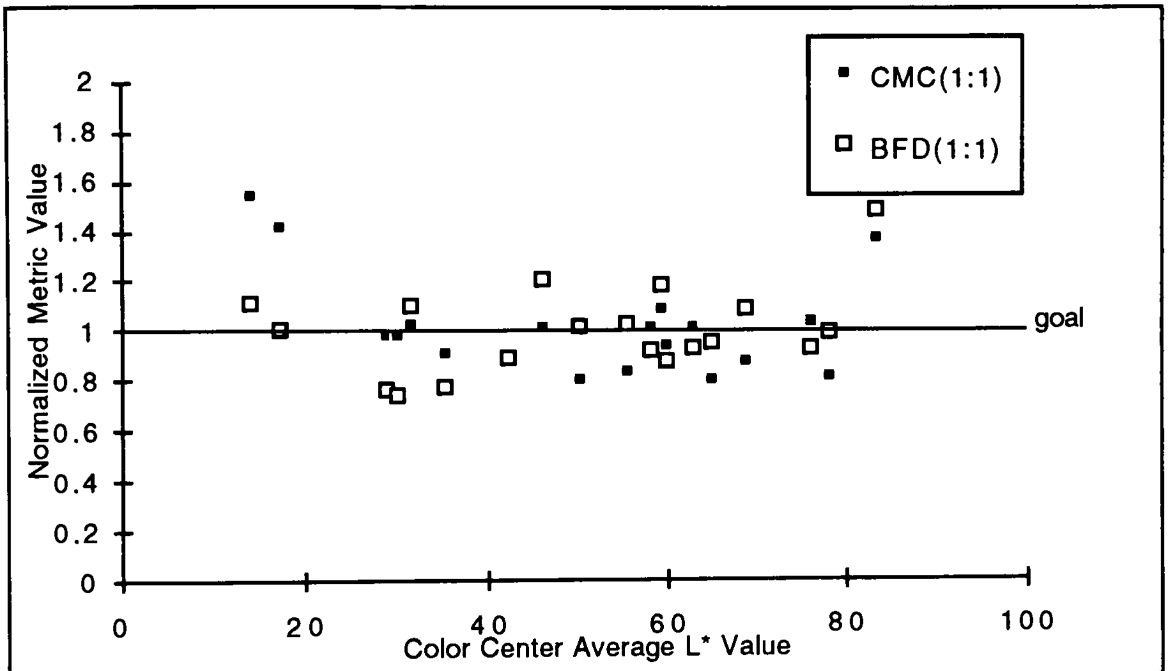

Figure 10: Comparison of Average Normalized CMC(1:1) and BFD(1:1) With Respect to Average Color Center L*

Figure 10 shows a comparison of the normalized CMC(1:1) and BFD(1:1) metrics corresponding to the pooled Phase I and II color-difference data with respect to increasing L*. Metrics were normalized as above and the average L* for each color center was calculated from Table A-III. Recall that the perfect industrial-sized color-difference metric, after normalization, would equal 1 for each of these samples. Thus, the goal value is 1. Qualitatively, Figure 10 demonstrates that for the most of the L* range, between L* values of approximately 25 and 80, CMC(1:1) and BFD(1:1) perform similarly, straddling the goal value. When L* values become very high, Figure 10 implies that CMC(1:1) and BFD(1:1) both tend to predict a larger color-difference than the observational data would indicate. For very small L* values, Figure 10 shows BFD(1:1) to have superior correlation to the Phase I and Phase II observers.

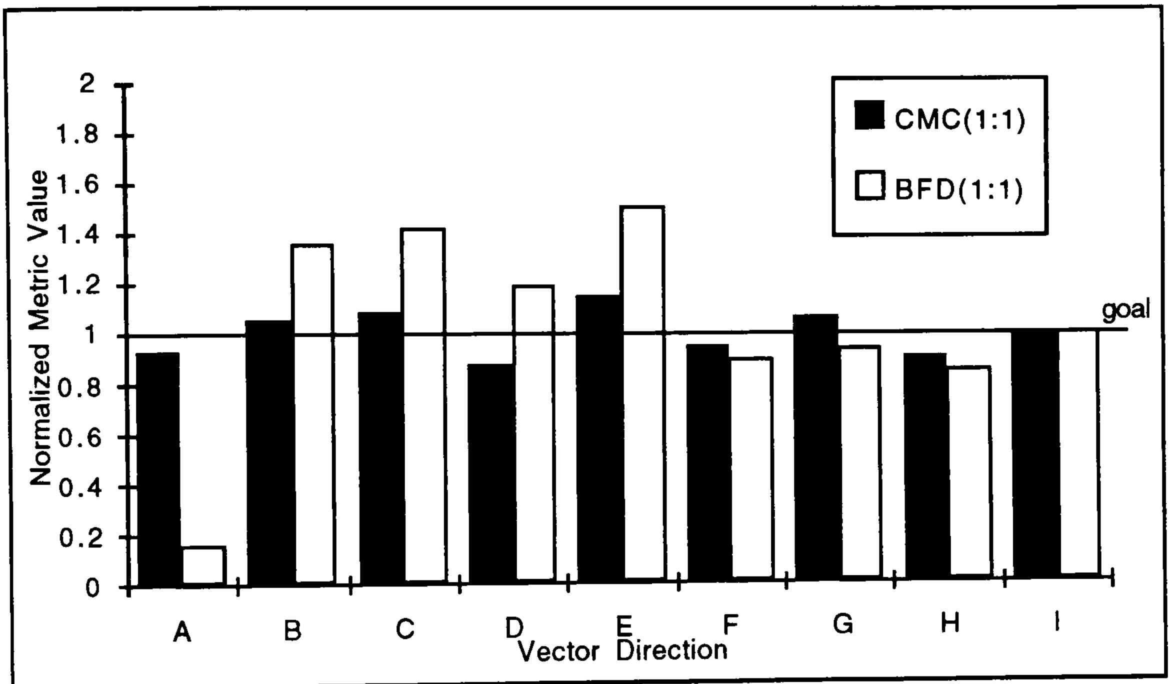

Figure 11: Comparison of Average Normalized CMC(1:1) and BFD(1:1) With Respect to Vector Direction

Figure 11 is another view of the same data. Normalized CMC(1:1) and BFD(1:1) are compared with respect to vector directions. Referring to Tables II and IV, vector direction A varies in only L*, vector directions B through E vary in chromaticness only and vector directions F through I vary in both lightness and chromaticness. Figure 11 demonstrates that CMC(1:1) and BFD(1:1) perform equally well for the latter category, but for lightness alone or chromaticness alone, CMC(1:1) performs better. In fact, for lightness discrimination, BFD(1:1) appears to be systematically underpredicting the response of the experimental observers to a large degree, explaining the source of the lower maxima in the BFD(1:1) curve in Figure 9.

Figures 10 and 11 demonstrate that in some ways BFD(1:1) did improve on CMC(1:1)'s performance, particularly for color-differences between very dark colors, and in other ways there was a degradation of performance. BFD(1:1)'s calculated discrimination metrics between colors that vary simultaneously in lightness and chromaticness were not adversely affected, but calculated differences between colors varying in either lightness or chromaticness alone did not perform as well as with CMC(1:1). Lightness differences were affected the most.

VII. Conclusions

In conclusion, it has been shown that the Phase II experiment consisted of more difficult tasks than the Phase I experiment due to the inclusion of color centers further from the anchor pair and vector directions which were generally more difficult to judge. This caused intra-observer noise to increase. A median filtering algorithm which was designed to have minimal effect upon the raw data was implemented in order to remove some of the most self-contradictory observer behavior. The approach did not affect Phase I T50's and had major impact, measured in tenths of single ΔE^*_{ab} unit, on approximately 10% of the Phase II T50's. The filtered T50's were compared to eight color difference formulae and CMC(1:1) was shown to match the results the best.

VIII. References

1 G. Snyder, "Visual Determination of Industrial Color-Difference Tolerance Using Probit Analyisis," M. S. thesis, Rochester Institute of Technology, Rochester, New York, 1991.

2 D. H. Alman, R. S. Berns, G. D. Snyder, W. A. Larsen, Performance Testing of Color-Difference Metrics Using a Color Tolerance Dataset, *Col. Res Appl.*, **14**, 139-151 (1989) .

3 L. Reniff, "Visual Determination of Color Differences Using Probit Analysis: Phase II," M. S. thesis, Rochester Institute of Technology, Rochester, New York, 1989.

4 R. Berns, D. Alman, L. Reniff, G. Snyder, M. Balonon-Rosen, Visual Determination of Suprathreshold Color-Difference Tolerances Using Probit Analysis, *Color Res. Appl.* **16**, 297-316 (1991).

5 D. Finney, Probit Analysis, 3rd Ed., Cambridge University Press, Cambridge, 1971.

6 A. Worthing, J. Geffner, Treatment of Experimental Data, John Wiley & Sons, New York, 1943, p.185, pp 216-235.

7 Roberson, A., The CIE 1976 Color-Difference Formulae, *Color Res. Appl.* **2**, 7-11 (1977).

8 CIE, International Commission on Illumination, Proceedings of the Eighth Session, Cambridge England, Bureau Central de la CIE, Paris, 1931.

9 Seim, T., A. Valberg, Towards a Uniform Color Space: A Better Formula to Describe the Munsell and OSA Color Scales, *Color Res. Appl.* **11**, 11-24 (1986).

10 G. Wyszecki, Recent Agreements Reached by the Colorimetry Committee of the Commission Internationale de l'Eclairage, *J. Opt. Soc. Am.* **58,** 290-292 (1986).

11 K. D. Chickering, FMC Color-Difference Formulas: Clarification Concerning Usage, *J. Opt. Soc. Am.* **61,** 118-122 (1971).

12 M. R. Luo and B. Rigg, BFD (l:c) Colour-difference Formula Part 1 Development of the formula, *J. Soc. Dyers and Colourists*, **103**, 86-94 (1987).

13 M. R. Luo and B. Rigg, BFD (l:c) Colour-difference Formula Part 2 Performance of the formula, *J. Soc. Dyers and Colourists*, **103**, 126-132 (1987).

14 F. J. J. Clarke, R. McDonald and B. Rigg, Modification to the JPC79 Colour-difference Formula, *J. Soc. Dyers and Colourists*, **100**, 128-132 (1984).

15 F. J. J. Clarke, R. McDonald and B. Rigg, Correspondence. Modification to the JPC79 Colour-difference Formula, *J. Soc. Dyers and Colourists*, **100**, 281-282 (1984).

16 R. S. Hunter, Photoelectric Tristimulus Colorimetry with Three Filters, NBS Circ. C429, 1942.

17 R. S. Hunter, R. W. Harold, The Measurement of Appearance, 2nd Edition, John Wiley and Sons, NY, 1987, pp 171-175, pp 119 194.

18 G. Wyszecki, Color appearance, in Handbook of Perception and Human Performance VI, K. R. Boff et al., Eds., Wiley, New York, 1986.

19 E. Coates, J. R. Provost, B. Rigg, The measurement and assessment of colour differences for industrial use. IV. The accuracy of visual assessments, *J. Soc. Dyers Colour.* **88**, 363-368 (1972).

20 D. Strocka, A. Brockes, and W. Paffhausen, Influence of experimental parameters on the evaluation of color-difference ellipsoids, *Color Res. Appl.* **8**, 169-175 (1983).

22 K. L. Kelly and D. B. Judd, Color: University Language and Dictionary of Names, *NBS Special Publication No. 440*, U.S. Government Printing Office, Washington, D.C., 1976.

23 A. Hardy, Handbook of Colorimetry, Technology Press, Cambridge, Ma, 1936 p. 9.

25 D. L. MacAdam, Color Measurement, Theme and Variations, 2nd Ed., Springer-Verlag, NY, 1985, p.129.

26 F. W. Billmeyer, Jr., Spectrophotometry, Color in Philately, R.H. White, Ed., The Philatelic Foundation, N.Y., 1979, p. 74-75.

27 S. S. Stevens, Measurement and Man, *Science*, **127**, 383-389 (1958).

28 A. H. Munsell, Munsell Book of Color, Munsell Color Co., Baltimore, Md., 1929-present.

29 A. H. Munsell, A Color Notation, Munsell Color Co., Baltimore, Md., 1936-1953.

30 A. H. Munsell, A Grammer of Color (1921), Faber Birren, Ed., Van Nostrand-Reinhold, NY, 1969.

31 D. Nickerson, History of the Munsell Color System and Its Scientific Application, *J. Opt. Soc. Am.*, **30**, 575-580 (1940); reprinted in Colro Res. Appl., **1**, 69-77 (1976).

32 D. B. Judd, Specification of Color Tolerances at the National Bureau of Standards, *Am. J. Psychol.*, **52**, p 418 (1939).

33 D. L. MacAdam, Visual Senitivities to Color Differences in Daylight, *J. Opt. Soc. Am.*, **32**, pp 247-274 (1942).

34 D. L. MacAdam, Specification of Small Chromaticity Differences, *J. Opt. Soc. Am.*, **33**, pp 18-26, (1943).

35 F. W. Billmeyer, Jr., M. Saltzman, Principles of Color Technology, 2nd Edition, John Wiley and Sons, NY, 1981, p 98.

36 CIE, International Commission on Illumination, Recommendations on Uniform Color Sapces, Color-Difference Equations, Psychometric Color Terms, Supplement No. 2 to CIE Publicatin No. 15, (E-1.3.1) 1971,/(TC-1.3) 1978, Bureau Central de la CIE, Paris, 1978.

37 F. W. Billmeyer, Jr., Current Status of the Color-Difference Problem, *J. Coat. Tech.* (1978).

38 R. G. Kuehni, *Color Res. Appl.*, **2**, p 187 (1977)

39 A. R. Robertson, CIE guidelines for coordinated research on colour-difference evaluation, *Color Res. Appl*, **3**, p 149 (1978)

40 W. D. Wright, The Measurement of Colour, 4th Edition, Van Nostrand Reinhold, NY, 1969, p 171

41 K. D. Chickering, Optimization of the MacAdam-Modified 1965 Friele Color Difference Formula, *J. Opt. Soc. Am.*, **57**, 537-541, (1967).

42 F. T. Simon, W. J. Goodwin, *Am Dyestuff Rep.*, **47**, p 105 (1958).

43 F. T. Simon, Small Color Differences Computation and Control, *Die Farbe*, **10**, pp 225-234 (1961).

44 K. McLaren, The Adams-Nickerson colour-difference formula, *J. Soc. Dye Colourists*, **86**, pp 354-366 (1970).

45 K. McLaren, Adams-Nickerson colour-difference formula, *J. Soc. Dye Colourists*, **87**, p 159 (1971)

46 G. Wyszecki, Proposal for a New Color-Difference Formula, *J. Opt. Soc Am.*, **53**, p 1318A (1963).

47 R. McDonald, Industrial pass/fail colour matching. Part III. Development of a pass/fail formula for use with instrumental measurement of colour difference. *J. Soc. Dye Colourists*, **96**, p 486 (1980).

48 McDonald, *J. Soc. Dye Colourists*, **97**. p 517 (1981).

49 S. M. Newell, D. Nickerson, and D. B. Judd, Final Report of the O.S.A. Subcomittee on the spacing of the Munsell Colors, J. *Opt. Soc. Am.*, **33**, 1943, pp 385-418

50 L. A. Riggs, Recollections of Early Laboratory Experiments on Vision, Foundations of Sensory Science, W. W. Dawson and J. M. Enoch Eds., Springer-Verlag, NY, 1984, pp 190 228

51 G. S. Brindley, Physiology of the Retina and Visual Pathway, Arnold, London, 1960

52 D. Nickerson, Tables for Use in Computing Small Color Differences, *Am. Dyest. Rep.*, **39** (1950).

53 L. Reniff, laboratory notes, private communication

54 SAS Institue Inc., SAS User's Guide: Statistic, Version 5 ed., SAS Institue Inc., Cary, NC, 1985.

55 R. Berns, personal communication, December 17, 1991.

56 *Standard Definitions of Terms Relating to Appearance of Materials*, ASTM Designation:E284, American Society for Testing and Materials, Philadelphia

57 A. R. Robertson, "Diagnostic Performance Evaluation of Spectrophotometers," Advances in Standards and Methodology in Spectrophotometry, C. Burgess and K.D. Mielinz Eds., Elsevier, New York, 1987

58 R. S. Berns and K.H.Petersen, Empirical Modeling of Systematic Spectrophotometric Errors, *Color Res. Appl.* **13**, 243(1988)

59 Rich, D.C., F.W. Billmeyer, Jr., Small and Moderate Color Differences. Color Difference-Perceptibility Ellipses Surface-Color Space, *Color Res. Appl.* **8**, 31-39 (1983)

62 S. M. Selby, Ed, Standard Mathematical Tables, Student Edition, 16th Ed., Chemical Rubber Co., Ohio, 1968, p 544.

63 D. C. Montgomery, Design and Analysis of Experiments, 2nd Ed., John Wiley & Sons, New York, 1984, p. 66.

64 S. Heckt, S. Shlaer, and M. H. Pirenne, Energy, quanta and vision,*J. Gen Physiol*, **25**: 819-840 (1942).

65 J. Larimer, D. H. Krantz and C. M. Cicerone, Opponent Process Additivity - II. Yellow/Blue Equilibria and Nonlinear Models, *Vision Research*, **15**, 732-731 (1975).

66 G. Milliken, D. Johnson, Analysis of Messy Data, Volume I: Designed Experiments, Van Nostrand Reinhold, 1984.

67 L. Sachs, Applied Statistics, A Handbook of Techniques, Springer-Verlag, New York, 1978.

73 D. Jameson and L. M. Hurvich, "Perceived Color and Its Dependence on Focal, Surrounding, and Preceding Stimulus Variables," *JOSA*, **49**, 890 - 898 (1959).

Appendix A: Tables

Table A-I: Comparing Current Frequency Data to Snyder's

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
Moderate blue	A						0.96	0.97	0.01
		1	0.26	0	0				
		2	0.5	10	10				
		3	0.75	16	16				
		4	1.01	28	27	-1			
		5	1.22	39	39				
		6	1.69	47	47				
		7	1.99	48	48				
	B						1.36	1.36	---
		8	0.53	1	1				
		9	0.82	5	5				
		10	1.14	16	16				
		11	1.4	31	31				
		12	1.75	40	41	1			
		13	1.96	46	46				
		14	2.35	47	47				
	C						1.53	1.54	0.01
		15	0.48	0	0				
		16	1.19	14	14				
		17	1.47	22	22				
		18	1.74	31	30	-1			
		19	1.96	41	41				
		20	2.21	47	47				
		21	2.45	49	50	1			
	D						1.11	1.12	0.01
		22	0.34	0	0				
		23	0.42	3	3				
		24	0.76	5	5				
		25	1.09	22	22				
		26	1.22	39	39				
		27	1.49	41	41				
		28	1.89	47	47				
	E						2.82	2.81	-0.01
		29	1.47	1	1				
		30	1.99	8	8				
		31	2.56	14	14				
		32	2.83	28	29	1			
		33	3.06	33	33				
		34	3.48	41	41				
		35	3.67	45	45				
		36	4.73	47	48	1			

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
Moderate greenish blue	A						0.79	0.8	0.01
		37	0.25	3	3				
		38	0.5	7	6	-1			
		39	0.75	25	25				
		40	0.97	36	36				
		41	1.21	47	47				
		42	1.56	47	47				
		43	1.8	50	50				
	B						1.61	1.62	0.01
		44	0.72	1	1				
		45	0.96	3	3				
		46	1.43	14	14				
		47	1.8	37	37				
		48	2.05	41	41				
		49	2.26	48	47	-1			
		50	2.33	48	48				
	C						1.63	1.61	-0.02
		51	0.76	0	0				
		52	1.21	10	10				
		53	1.47	17	17				
		54	1.75	27	28	1			
		55	1.95	46	46				
		56	2.44	48	48				
		57	2.95	48	50	2			
	D						1.80	1.79	-0.01
		58	1	3	3				
		59	1.35	13	13				
		60	1.56	17	17				
		61	1.85	22	23	1			
		62	2.12	44	44				
		63	2.62	44	44				
		64	3.18	48	49	1			
	E						1.49	1.51	0.02
		65	0.73	3	3				
		66	0.98	8	8				
		67	1.57	33	31	-2			
		68	1.93	38	38				
		69	2.32	46	46				
		70	3.06	49	49				
		71	3.12	50	50				
Medium gray	A						0.93	0.94	0.01
		72	0.24	1	1				
		73	0.48	2	2				
		74	0.72	9	8	-1			
		75	0.96	27	27				

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
		76	1.29	46	46				
		77	1.72	50	50				
		78	1.99	50	50				
	B						0.89	0.89	---
		79	0.24	0	0				
		80	0.52	5	5				
		81	0.76	15	15				
		82	1	37	38	1			
		83	1.21	44	44				
		84	1.61	49	49				
		85	1.83	49	49				
	C						1.32	1.32	---
		86	0.74	2	3	1			
		87	1.01	17	17				
		88	1.56	33	33				
		89	2.09	47	47				
		90	2.17	49	49				
		91	2.54	50	50				
		92	3.08	50	50				
	D						0.92	0.91	-0.01
		93	0.48	1	1				
		94	0.71	9	12	3			
		95	0.96	30	30				
		96	1.2	43	43				
		97	1.43	49	49				
		98	1.66	50	50				
		99	2.19	50	50				
	E						1.29	1.3	0.01
		100	0.9	6	6				
		101	1.18	19	19				
		102	1.4	32	31	-1			
		103	1.91	47	47				
		104	2.29	50	50				
		105	2.43	50	50				
		106	2.81	50	50				
Moderate bluish green	A						0.94	0.95	0.01
		107	0.3	2	2				
		108	0.53	4	4				
		109	0.74	10	10				
		110	0.98	30	30				
		111	1.26	40	39	-1			
		112	1.47	49	49				
		113	2.03	50	50				
	B						2.28	2.19	-0.09*
		114	0.82	0	0				

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
		115	1.49	1	3	2			
		116	1.69	4	4				
		117	1.96	16	17	1			
		118	2.2	22	23	1			
		119	2.41	28	39	11*			
		120	2.93	46	46				
		121	3.68	50	50				
	C						1.31	1.3	-0.01
		122	0.56	0	0				
		123	0.78	4	4				
		124	1.04	14	14				
		125	1.24	24	23	-1			
		126	1.51	35	36	1			
		127	1.81	42	43	1			
		128	2.09	49	50	1			
	D						1.67	1.66	-0.01
		129	0.57	0	0				
		130	0.98	1	1				
		131	1.25	5	5				
		132	1.46	17	17				
		133	1.71	24	25	1			
		134	2	40	40				
		135	2.13	47	48	1			
	E						1.77	1.77	---
		136	1.06	2	2				
		137	1.22	4	4				
		138	1.53	13	13				
		139	1.75	22	22				
		140	1.93	35	35				
		141	2.15	42	42				
		142	2.54	48	48				
Light brown	A						0.92	0.9	-0.02
		143	0.27	2	2				
		144	0.49	2	3	1			
		145	0.73	17	18	1			
		146	0.96	23	23				
		147	1.26	47	47				
		148	1.53	48	48				
		149	1.93	49	50	1			
	B						1.39	1.37	-0.02
		150	0.47	0	1	1			
		151	0.78	1	1				
		152	1.24	16	18	2			
		153	1.53	38	37	-1			
		154	1.83	43	43				
		155	2.03	48	47	-1			

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfiltred Freq. of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfiltred T50	Snyder T50	T50 diff btwn current and Snyder
		156	2.31	50	50				
	C						1.46	1.46	---
		157	0.54	2	2				
		158	0.98	5	5				
		159	1.33	14	14				
		160	1.5	25	26	1			
		161	1.83	45	45				
		162	2.01	46	46				
	D						1.59	1.6	0.01
		163	0.7	1	1				
		164	0.96	2	2				
		165	1.22	14	14				
		166	1.75	33	33				
		167	1.99	39	39				
		168	2.25	45	45				
		169	3.03	50	50				
	E						1.13	1.13	---
		170	0.24	0	0				
		171	0.86	5	5				
		172	1.08	19	19				
		173	1.25	37	37				
		174	1.45	46	46				
		175	2.01	50	50				
		176	2.21	50	50				
Grayish purple	A						0.95	0.95	---
		177	0.24	0	0				
		178	0.75	15	15				
		179	1.23	42	43	1			
		180	1.49	46	46				
		181	1.7	48	48				
		182	1.99	50	50				
	B						1.47	1.48	0.01
		183	0.54	0	0				
		184	1.03	2	2				
		185	1.28	16	16				
		186	1.48	31	31				
		187	1.82	41	40	-1			
		188	2.06	46	46				
		189	2.26	50	50				
	C						1.41	1.4	-0.01
		190	0.74	0	0				
		191	1.04	3	3				
		192	1.21	9	10	1			
		193	1.44	36	37	1			
		194	1.82	45	46	1			
		195	1.99	47	48	1			

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
	D						1.22	1.22	---
		196	0.7	2	2				
		197	0.95	8	8				
		198	1.14	22	23	1			
		199	1.42	37	37				
		200	1.78	48	49	1			
		201	2.01	49	49				
	E						2.87	2.84	-0.03
		202	1.25	0	0				
		203	1.49	0	0				
		204	1.73	2	2				
		205	2.02	2	2				
		206	2.23	4	4				
		207	2.44	14	14				
		208	2.85	25	26	1			
		209	3.04	33	34	1			
		210	3.91	47	48	1			
Dark reddish orange	A						0.94	0.95	0.01
		211	0.35	2	2				
		212	0.49	4	4				
		213	0.76	18	19	1			
		214	1	25	25				
		215	1.26	44	43	-1			
		216	1.52	47	46	-1			
		217	2.01	49	49				
	B						1.94	1.94	---
		218	0.75	0	0				
		219	1.26	1	1				
		220	1.5	5	5				
		221	1.75	15	15				
		222	2.02	24	24				
		223	2.15	42	43	1			
		224	2.49	46	46				
		225	2.97	50	50				
	C						1.54	1.55	0.01
		226	0.5	0	0				
		227	0.99	4	4				
		228	1.24	20	20				
		229	1.49	26	26				
		230	1.8	29	28	-1			
		231	2	43	42	-1			
		232	2.54	48	48				
	D						2.01	1.99	-0.02
		233	1.14	3	4	1			
		234	1.47	9	9				

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
		235	1.79	18	17	-1			
		236	1.98	27	27				
		237	2.23	37	37				
		238	2.54	36	39	3			
		239	2.96	45	45				
	E						1.31	1.33	0.02
		240	0.26	1	1				
		241	1.02	12	12				
		242	1.23	16	16				
		243	1.51	37	37				
		244	1.77	43	43				
		245	2.04	48	48				
		246	2.56	50	50				
Moderate yellow	A						1.18	1.18	---
		247	0.46	2	2				
		248	0.74	6	6				
		249	0.98	11	11				
		250	1.25	29	29				
		251	1.48	42	42				
		252	1.78	46	46				
		253	2.08	50	50				
	B						1.44	1.45	0.01
		254	0.64	2	2				
		255	0.91	4	4				
		256	1.22	13	13				
		257	1.52	27	27				
		258	1.74	40	40				
		259	2.03	47	47				
		260	2.39	49	49				
	C						2.21	2.19	-0.02
		261	0.76	0	0				
		262	1.03	0	0				
		263	1.22	2	1	-1			
		264	1.75	11	12	1			
		265	2.08	23	24	1			
		266	2.28	25	25				
		267	2.5	36	37	1			
		268	2.84	44	45	1			
		269	3.17	47	48	1			
	D						1.62	1.61	-0.01
		270	0.82	1	1				
		271	1.04	4	4				
		272	1.28	6	6				
		273	1.5	16	16				
		274	1.72	32	33	1			

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE*ab	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
		275	2.08	45	46	1			
	E						1.29	1.29	---
		276	0.72	1	1				
		277	0.95	6	6				
		278	1.17	16	16				
		279	1.44	36	36				
		280	1.74	46	46				
		281	1.94	50	50				
		282	2.64	50	50				
Grayish yellow green	A						0.86	0.83	-0.03
		283	0.28	2	2				
		284	0.47	3	3				
		285	0.74	24	25	1			
		286	1	35	36	1			
		287	1.26	43	44	1			
		288	1.52	48	49	1			
		289	2	49	50	1			
	B						1.15	1.14	-0.01
		290	0.5	2	2				
		291	0.8	4	4				
		292	1.02	19	19				
		293	1.29	34	36	2			
		294	1.5	42	42				
		295	1.68	49	49				
		296	1.96	48	49	1			
	C						1.43	1.44	0.01
		297	0.66	1	1				
		298	1.02	9	9				
		299	1.27	14	14				
		300	1.54	26	26				
		301	1.76	39	39				
		302	1.98	50	50				
		303	2.42	50	50				
	D						1.20	1.21	0.01
		304	0.71	1	1				
		305	0.99	13	13				
		306	1.18	21	22	1			
		307	1.51	45	44	-1			
		308	1.76	48	48				
		309	1.95	50	50				
		310	2.17	50	50				
	E						1.69	1.7	0.01
		311	0.82	1	1				
		312	1.51	13	13				
		313	1.77	31	31				

Table A-I (Cont.):

Color Center	Vector Set	Snyder Sample #	ΔE^*_{ab}	Current Unfltred Freq.of Rejectn.	Snyder Freq. of Rejectn.	Freq. diff btwn current and Snyder	Current Unfltred T50	Snyder T50	T50 diff btwn current and Snyder
		314	1.96	39	38	-1			
		315	2.37	48	48				
		316	2.53	48	47	-1			
		317	3.25	50	50				

*Frequency difference of 11 for sample number 119 (Moderate bluish green vector B) is considered to be the result of a typographical error made during the reporting of Phase I by Snyder. Frequency of rejection should probably have been 29, thus frequency difference would rightly have been 1. The T50 value reported for the current frequencies of Moderate bluish green vector B is far more believable than Snyder's.

Table A-II: Comparing Current Frequency Data to Reniff's

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
Light bluish green	A						1.20	1.20	---
		1	1.01	8	8				
		2	1.19	36	36				
		3	1.53	37	37				
		4	2.09	38	38				
		5	2.45	40	40				
	B						2.38	2.38	---
		6	1.90	17	17				
		7	2.23	20	20				
		8	2.50	25	25				
		9	3.00	41	41				
		10	3.99	47	47				
	C						1.78	1.79	0.01
		11	1.47	15	14	1			
		12	1.17	7	7				
		13	1.69	22	22				
		14	2.00	36	36				
		15	2.51	42	42				
	F						1.29	1.29	---
		16	0.59	4	4				
		17	1.05	9	9				
		18	1.36	30	30				
		19	1.45	36	36				
		20	1.95	46	46				
		21	2.43	49	49				
	G						1.69	1.69	---
		22	1.24	3	3				
		23	1.56	16	16				
		24	1.67	34	34				
		25	2.01	37	37				
		26	2.38	47	48	-1			
	H						1.71	1.71	---
		27	1.29	17	17				
		28	1.63	13	13				
		29	1.74	40	40				
		30	1.98	29	29				
		31	2.57	36	36				
	I						1.47	1.47	---
		32	1.15	11	11				
		33	1.30	16	16				
		34	1.68	42	42				
		35	1.82	32	32				
		36	1.98	45	45				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
Brilliant greenish blue	A						1.24	1.23	-0.01
		37	0.47	4	4				
		38	0.76	25	25				
		39	0.95	29	29				
		40	1.27	20	19	1			
		41	1.60	23	24	-1			
		42	1.88	41	41				
	B						1.51	1.47	-0.04
		43	1.26	17	18	-1			
		44	1.68	32	32				
		45	1.78	35	35				
		46	2.15	22	23	-1			
		47	2.23	37	38	-1			
	C						2.05	2.05	---
		48	1.42	11	11				
		49	1.70	16	16				
		50	2.18	31	31				
		51	2.60	39	39				
		52	2.82	37	38	-1			
	F						1.02	1.01	-0.01
		53	0.76	7	8	-1			
		54	0.99	34	34				
		55	1.43	47	47				
		56	1.69	33	33				
		57	2.09	50	50				
	G						1.62	1.62	---
		58	1.30	18	18				
		59	1.56	16	16				
		60	1.72	35	35				
		61	2.04	34	34				
		62	2.64	46	46				
	H						1.37	1.38	0.01
		63	0.98	8	7	1			
		64	1.37	31	31				
		65	1.40	16	16				
		66	1.66	40	40				
		67	1.56	38	38				
		68	2.37	48	48				
	I						2.07	2.07	---
		69	1.74	23	23				
		70	2.12	24	24				
		71	2.24	17	17				
		72	2.46	36	36				
		73	2.64	46	46				
		74	2.89	41	41				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
Moderate purplish pink	A						1.34	1.33	-0.01
		75	0.89	4	4				
		76	1.04	20	20				
		77	1.24	28	28				
		78	1.56	29	30	-1			
		79	1.72	35	35				
		80	1.87	43	43				
	B						2.41	2.41	---
		81	1.47	7	7				
		82	1.94	12	12				
		83	2.26	18	18				
		84	2.38	36	36				
		85	2.62	27	27				
		86	2.94	34	34				
	C						1.94	1.96	0.02
		87	1.45	14	14				
		88	1.82	23	22	1			
		89	2.04	25	24	1			
		90	2.11	28	28				
		91	2.53	41	40	1			
	F						1.70	1.70	---
		92	1.15	21	21				
		93	1.30	12	12				
		94	1.59	26	26				
		95	1.76	15	15				
		96	2.01	38	38				
	G						1.58	1.55	-0.03
		97	0.50	2	2				
		98	0.62	1	1				
		99	0.83	8	8				
		100	0.97	5	5				
		101	0.96	24	24				
		102	1.51	13	13				
		103	1.75	34	36	-2			
	H						1.59	1.59	---
		104	0.82	5	5				
		105	1.18	4	4				
		106	1.33	18	18				
		107	1.62	28	28				
		108	1.92	37	37				
		109	2.15	43	43				
	I						1.72	1.73	0.01
		110	0.89	2	2				
		111	1.33	13	13				
		112	1.46	19	19				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		113	1.82	36	36				
		114	2.37	37	37				
Light gray	A						1.26	1.26	---
		115	1.09	16	16				
		116	1.31	34	34				
		117	1.54	29	29				
		118	1.96	39	39				
		119	2.14	44	44				
		120	2.30	42	42				
	B						0.99	0.99	---
		121	0.96	22	22				
		122	1.09	30	30				
		123	1.23	30	30				
		124	1.49	42	42				
		125	2.04	46	46				
	C						1.80	1.80	---
		126	1.34	11	11				
		127	1.81	29	29				
		128	2.11	29	29				
		129	2.17	38	38				
		130	2.78	47	47				
		131	2.81	47	47				
	F						1.24	1.24	---
		132	0.88	9	9				
		133	0.95	20	20				
		134	1.20	27	27				
		135	1.41	30	30				
		136	1.72	34	34				
		137	1.86	45	45				
	G						1.36	1.37	0.01
		138	0.95	13	13				
		139	1.26	22	22				
		140	1.38	28	28				
		141	1.62	32	32				
		142	1.88	37	37				
	H						1.02	1.02	---
		143	0.52	4	4				
		144	0.82	14	14				
		145	1.05	21	21				
		146	1.19	38	38				
		147	1.66	49	49				
	I						1.56	1.56	---
		148	0.93	11	11				
		149	1.40	19	19				
		150	1.58	27	27				
		151	1.84	29	29				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		152	2.01	39	39				
Strong orange yellow	A						0.34	0.66	0.32
		153	1.42	23	23				
		154	1.54	32	32				
		155	1.75	27	27				
		156	1.84	32	32				
		157	2.03	36	36				
		158	2.28	25	26	-1			
	B						1.80	1.80	---
		159	1.39	21	21				
		160	1.68	22	22				
		161	2.08	25	25				
		162	2.34	31	31				
		163	2.65	46	46				
	C						5.13	5.11	-0.02
		164	2.79	7	7				
		165	3.17	6	6				
		166	3.32	7	6	1			
		167	3.55	6	6				
		168	3.78	13	13				
		169	3.98	15	15				
	F						1.97	1.97	---
		170	1.62	17	17				
		171	1.85	31	31				
		172	2.05	22	22				
		173	2.42	30	30				
		174	2.85	38	38				
		175	3.76	46	46				
	G						2.14	2.15	0.01
		176	1.35	5	5				
		177	1.41	16	16				
		178	1.94	35	35				
		179	2.05	32	32				
		180	2.33	17	17				
		181	2.40	26	26				
	H						1.69	1.69	---
		182	1.23	19	19				
		183	1.61	18	18				
		184	2.05	34	34				
		185	2.43	39	39				
		186	3.42	46	45	1			
	I						2.02	2.02	---
		187	1.58	17	17				
		188	2.14	17	17				
		189	2.35	43	43				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		190	2.35	31	31				
		191	2.61	39	39				
		192	2.94	40	40				
Black	A						1.21	1.21	---
		193	0.74	13	13				
		194	0.88	20	20				
		195	1.35	17	17				
		196	1.51	37	37				
		197	1.73	42	42				
	B						0.78	0.78	---
		198	0.47	7	7				
		199	0.63	16	16				
		200	0.70	25	25				
		201	0.97	39	39				
		202	1.15	39	39				
	C						1.30	1.30	---
		203	0.85	11	11				
		204	1.08	24	24				
		205	1.26	25	24	1			
		206	1.46	30	30				
		207	1.53	26	26				
		208	2.01	43	43				
	F						1.08	1.08	---
		209	0.44	10	10				
		210	0.87	15	15				
		211	1.09	20	20				
		212	1.30	43	43				
		213	1.73	38	38				
	G						1.06	1.05	-0.01
		214	0.65	8	8				
		215	0.89	23	23				
		216	1.20	27	27				
		217	1.23	34	34				
		218	1.45	39	39				
	H						1.02	1.02	---
		219	0.69	10	10				
		220	0.78	23	23				
		221	0.84	16	16				
		222	1.10	25	25				
		223	1.25	41	41				
		224	1.58	38	38				
	I						0.94	0.95	0.01
		225	0.65	8	8				
		226	0.77	22	22				
		227	1.02	33	33				
		228	1.23	33	33				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		229	1.39	42	42				
Moderate reddish brown	A						0.99	0.99	---
		230	0.61	19	19				
		231	0.80	13	13				
		232	1.17	38	38				
		233	1.31	38	38				
		234	1.56	26	26				
	B						1.15	1.16	0.01
		235	0.78	18	18				
		236	0.97	12	12				
		237	1.10	24	24				
		238	1.36	33	33				
		239	1.50	35	35				
		240	2.11	48	48				
	C						1.57	1.55	-0.02
		241	1.01	7	7				
		242	1.25	11	11				
		243	1.45	24	25	-1			
		244	1.78	36	36				
		245	1.95	35	36	-1			
	F						1.06	1.06	---
		246	0.90	23	23				
		247	1.01	28	28				
		248	1.22	24	24				
		249	1.34	19	19				
		250	1.57	46	46				
		251	1.95	35	35				
	G						0.95	0.93	-0.02
		252	0.62	8	8				
		253	0.74	20	21	-1			
		254	0.93	14	14				
		255	1.10	42	43	-1			
		256	1.27	38	39	-1			
		257	1.54	47	48	-1			
	H						1.38	1.39	0.01
		258	0.62	5	5				
		259	1.01	23	23				
		260	1.29	13	13				
		261	1.54	28	28				
		262	1.83	41	41				
	I						0.91	0.91	---
		263	0.60	11	11				
		264	0.84	20	20				
		265	0.93	25	25				
		266	1.12	39	39				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		267	1.32	41	41				
Dark bluish green	A						0.97	0.97	---
		268	0.78	15	15				
		269	1.03	32	32				
		270	1.33	29	29				
		271	1.35	44	44				
		272	1.63	38	38				
	B						3.55	3.07	-0.48
		273	2.75	14	19	-5			
		274	3.58	23	28	-5			
		275	4.07	40	45	-5			
		276	4.21	31	36	-5			
		277	5.43	38	43	-5			
	C						1.40	1.42	0.02
		278	0.86	11	11				
		279	1.15	21	21				
		280	1.37	18	18				
		281	1.62	33	33				
		282	1.69	34	34				
		283	1.96	39	38	1			
	F						2.20	2.20	---
		284	1.37	12	12				
		285	1.70	14	14				
		286	2.42	23	23				
		287	2.77	41	41				
		288	3.71	47	47				
	G						1.35	1.36	0.01
		289	0.66	4	4				
		290	0.97	11	11				
		291	1.10	11	11				
		292	1.32	18	18				
		293	1.43	26	26				
		294	1.77	47	47				
	H						1.43	1.43	---
		295	1.08	13	13				
		296	1.23	13	13				
		297	1.44	42	42				
		298	1.68	36	36				
		299	2.15	26	26				
		300	2.24	39	38	1			
	I						1.19	1.19	---
		301	0.65	5	5				
		302	1.10	30	30				
		303	1.44	41	41				
		304	1.75	34	34				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		305	2.19	39	39				
Very dark red	A						1.01	1.02	0.01
		306	0.75	12	12				
		307	0.98	38	38				
		308	1.12	28	28				
		309	1.29	23	23				
		310	1.53	36	36				
	B						1.69	1.70	0.01
		311	1.19	14	14				
		312	1.54	14	14				
		313	1.88	29	29				
		314	2.18	43	43				
		315	2.29	42	42				
	C						1.71	1.69	-0.02
		316	0.72	4	4				
		317	1.26	12	12				
		318	1.57	21	21				
		319	1.72	32	32				
		320	1.98	28	28				
	F						1.45	1.45	---
		321	0.67	5	5				
		322	1.09	12	12				
		323	1.09	16	16				
		324	1.47	14	14				
		325	1.62	40	40				
	G						1.65	1.65	---
		326	0.98	7	7				
		327	1.28	20	20				
		328	1.46	29	29				
		329	1.75	22	22				
		330	1.96	26	26				
		331	2.28	42	42				
	H						1.80	1.81	0.01
		332	1.33	13	13				
		333	1.62	18	18				
		334	1.74	24	23	1			
		335	2.05	32	32				
		336	2.14	32	32				
		337	2.32	42	42				
	I						1.30	1.30	---
		338	0.76	8	8				
		339	0.98	12	12				
		340	1.29	33	33				
		341	1.56	29	29				
		342	1.90	43	43				
Dark blue	A						1.01	1.01	---

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		343	0.70	12	12				
		344	0.94	24	24				
		345	1.04	29	29				
		346	1.33	34	34				
		347	1.47	42	42				
		348	1.59	41	41				
	B						1.40	1.40	---
		349	1.15	13	13				
		350	1.31	26	26				
		351	1.78	39	39				
		352	2.38	45	45				
		353	2.69	46	46				
	C						1.18	1.17	-0.01
		354	0.48	1	1				
		355	0.82	11	11				
		356	1.11	24	25	-1			
		357	1.52	43	43				
		358	2.09	46	46				
	F						0.93	0.93	---
		359	0.77	16	16				
		360	1.00	25	25				
		361	1.11	35	35				
		362	1.28	45	45				
		363	1.51	33	33				
		364	1.77	46	46				
	G						1.69	1.69	---
		365	1.07	7	6	1			
		366	1.40	26	27	-1			
		367	1.75	21	21				
		368	1.91	27	27				
		369	2.29	43	43				
	H						1.04	1.05	0.01
		370	0.51	8	8				
		371	0.92	11	11				
		372	0.98	25	25				
		373	1.08	39	39				
		374	1.29	30	30				
		375	1.44	37	35	2			
	I						1.37	1.27	-0.10
		376	1.45	26	27	-1			
		377	1.68	24	25	-1			
		378	1.73	33	32	1			
		379	1.97	19	19				
		380	2.19	38	38				
		381	2.51	30	30				
Moderate blue	F						1.01	1.00	-0.01

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		382	0.96	20	20				
		383	1.07	28	28				
		384	1.31	42	42				
		385	1.60	42	42				
		386	1.67	46	46				
	G						1.13	1.13	---
		387	0.91	29	29				
		388	1.01	26	26				
		389	1.19	10	10				
		390	1.29	26	26				
		391	1.53	38	38				
	H						0.87	0.87	---
		392	0.59	6	6				
		393	0.80	30	30				
		394	0.94	31	31				
		395	1.12	34	34				
		396	1.57	46	46				
	I						1.73	1.73	---
		397	1.12	6	6				
		398	1.48	9	9				
		400	1.84	35	35				
		401	2.03	36	36				
		402	2.51	48	48				
Moderate greenish blue	F						1.18	1.19	0.01
		403	1.00	21	21				
		404	1.11	22	22				
		405	1.31	22	22				
		406	1.57	46	46				
		407	1.79	44	44				
	G						1.00	1.00	---
		408	0.68	6	6				
		409	0.90	22	22				
		410	1.08	35	35				
		411	1.31	34	34				
		412	1.40	45	45				
	H						1.20	1.20	---
		413	0.81	9	9				
		414	1.09	16	16				
		415	1.20	32	32				
		416	1.37	42	42				
		417	1.60	29	29				
	I						0.86	0.87	0.01
		418	0.72	14	14				
		419	0.93	29	29				
		420	1.04	46	46				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		421	1.17	37	37				
		422	1.33	32	32				
		423	0.06	2	2				
Moderate bluish green	F						1.28	1.28	---
		424	0.96	14	14				
		425	1.24	23	23				
		426	1.43	32	32				
		427	1.51	29	29				
		428	1.79	47	47				
	G						1.47	1.47	---
		429	0.92	6	6				
		430	1.20	3	3				
		431	1.54	33	33				
		432	1.59	29	29				
		433	1.77	46	46				
		434	2.14	46	46				
	H						1.29	1.29	---
		435	0.84	16	16				
		436	1.04	15	15				
		437	1.16	23	23				
		438	1.36	20	20				
		439	1.62	32	32				
		440	1.77	45	45				
	I						1.15	1.15	---
		441	0.92	13	13				
		442	1.16	28	28				
		443	1.49	45	45				
		444	1.42	32	32				
		445	1.54	44	44				
Medium gray	F						1.01	1.01	---
		446	0.46	4	4				
		447	0.88	21	21				
		448	1.07	21	21				
		449	1.20	29	29				
		450	1.29	43	43				
		451	1.37	46	46				
	G						1.02	1.03	0.01
		452	0.60	2	2				
		453	0.75	12	12				
		454	1.30	42	42				
		455	1.23	37	37				
		456	1.44	47	47				
		457	1.68	47	46	1			
	H						0.87	0.87	---
		458	0.59	5	5				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE^*_{ab}	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		459	0.78	14	14				
		460	0.88	25	24	1			
		461	1.02	44	44				
		462	1.22	45	45				
	I						0.94	0.94	---
		463	0.57	5	5				
		464	0.80	10	10				
		465	0.93	36	36				
		466	1.04	32	32				
		467	1.27	38	38				
Light brown	F						1.45	1.46	0.01
		468	1.27	14	14				
		469	1.42	29	29				
		470	1.69	31	31				
		471	1.82	37	37				
		472	1.97	45	45				
		473	2.34	45	45				
	G						1.08	1.09	0.01
		474	0.70	6	6				
		475	0.87	10	10				
		476	0.95	18	18				
		477	1.20	38	38				
		478	1.63	45	45				
	H						1.17	1.17	---
		479	0.67	4	4				
		480	0.88	14	14				
		481	1.20	34	34				
		482	1.36	30	30				
		483	1.56	38	38				
	I						1.02	1.02	---
		484	0.62	8	8				
		485	0.80	12	12				
		486	0.94	25	25				
		487	1.20	38	38				
		488	1.42	38	38				
Grayish purple	F						1.06	1.06	---
		489	0.69	15	15				
		490	0.76	7	7				
		491	0.92	16	16				
		492	1.01	26	26				
		493	1.19	31	31				
	G						1.40	1.40	---
		494	1.08	16	16				
		495	1.19	18	18				
		496	1.31	24	24				
		497	1.51	21	21				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		498	1.77	41	41				
	H						1.02	1.02	---
		499	0.84	14	13	1			
		500	0.99	29	29				
		501	1.04	29	29				
		502	1.19	28	28				
		503	1.27	35	35				
	I						1.38	1.38	---
		504	0.64	13	13				
		505	0.96	14	14				
		506	1.08	23	23				
		507	1.36	27	27				
		508	1.38	32	32				
		509	1.58	21	21				
Dark reddish orange	F						1.65	1.65	---
		510	1.30	8	8				
		511	1.44	18	18				
		512	1.55	26	26				
		513	1.66	33	33				
		514	1.92	43	43				
		515	2.25	27	27				
	G						0.39	0.40	0.01
		516	1.03	26	26				
		517	1.19	16	16				
		518	0.09	28	28				
		519	1.48	30	30				
		520	1.67	38	38				
	H						1.57	1.57	---
		521	1.29	22	22				
		522	1.43	8	8				
		523	1.53	15	15				
		524	1.59	32	32				
		525	1.90	42	42				
	I						1.18	1.17	-0.01
		526	1.15	28	28				
		527	1.19	21	21				
		528	1.41	33	33				
		529	1.53	36	36				
		530	1.62	45	45				
		531	1.91	45	45				
Moderate yellow	F						1.45	1.45	---
		532	1.12	14	14				
		533	1.56	27	27				
		534	1.69	36	36				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		535	1.88	37	37				
		536	2.13	46	46				
	G						1.39	1.40	0.01
		537	1.00	12	12				
		538	1.27	23	23				
		539	1.49	30	30				
		540	1.59	28	28				
		541	1.87	37	37				
		542	1.93	43	43				
	H						1.27	1.27	---
		543	0.90	8	8				
		544	1.20	30	30				
		545	1.29	21	21				
		546	1.44	31	31				
		547	1.81	44	44				
	I						1.16	1.16	---
		548	0.79	4	4				
		549	0.99	21	21				
		550	1.27	38	38				
		551	1.37	27	27				
		552	1.59	44	44				
Grayish yellow green	F						0.80	0.80	---
		553	0.94	7	7				
		554	0.04	21	21				
		555	1.20	36	36				
		556	1.48	45	45				
		557	1.66	33	33				
	G						1.32	1.32	---
		558	1.23	20	20				
		559	1.39	23	23				
		560	1.49	35	35				
		561	1.82	37	37				
		562	2.25	38	38				
	H						1.06	1.07	0.01
		563	0.78	19	18	1			
		564	0.89	9	9				
		565	1.08	29	29				
		566	1.27	34	34				
		567	1.50	36	36				
		568	1.61	49	49				
	I						1.00	1.01	0.01
		569	0.65	5	5				
		570	0.80	22	22				
		571	1.03	16	16				
		572	1.27	43	43				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		573	1.37	42	42				
		574	1.51	48	48				
Moderate blue	D						0.99	0.99	---
		575	0.74	13	13				
		576	1.13	31	31				
		577	1.22	35	35				
		578	1.50	46	46				
		579	1.93	47	47				
	E						3.31	3.27	-0.04
		580	2.11	4	4				
		581	2.60	12	12				
		582	2.82	27	27				
		583	3.01	18	18				
		584	3.72	29	30	-1			
Moderate greenish blue	A						1.01	1.02	0.01
		585	0.59	4	4				
		586	0.69	25	25				
		587	0.76	14	13	1			
		588	0.90	21	21				
		589	1.28	33	33				
	D						1.55	1.56	0.01
		590	1.02	4	4				
		591	1.35	25	25				
		592	1.55	27	27				
		593	1.90	33	33				
		594	2.11	42	42				
Medium gray	B						0.87	0.88	0.01
		595	0.52	7	7				
		596	0.76	18	18				
		597	0.91	27	27				
		598	0.94	31	31				
		599	1.21	41	41				
Light brown	C						1.61	1.61	---
		600	0.93	4	4				
		601	1.37	20	20				
		602	1.55	21	21				
		603	1.85	32	32				
		604	1.95	38	38				
Grayish purple	C						1.71	1.71	---
		605	1.01	2	2				
		606	1.22	8	8				
		607	1.44	17	17				
		608	1.23	16	16				
		609	1.65	22	22				

Table A-II (Cont.):

Color Center	Vector Set	Reniff Sample #	ΔE*ab	Current Unfltred Freq. of Rejectn.	Reniff Freq. of Rejectn.	Freq. diff btwn current and Reniff	Current Unfltred T50	Reniff T50	T50 diff btwn current and Reniff
		610	1.88	30	30				
Dark reddish orange	C						1.70	1.70	---
		611	1.04	5	5				
		612	1.35	12	12				
		613	1.42	17	17				
		615	1.77	24	24				
		616	1.93	36	36				
Moderate bluish green	A						1.03	1.03	---
		617	0.29	6	6				
		618	0.83	14	14				
		619	0.84	17	17				
		620	0.97	26	26				
		621	1.18	31	31				
	B						2.58	2.59	0.01
		622	1.60	8	8				
		623	2.03	14	14				
		624	2.21	17	17				
		625	2.43	23	23				
		626	2.94	32	32				
	C						1.22	1.23	0.01
		627	0.81	8	8				
		628	1.03	19	19				
		629	1.23	25	25				
		630	1.56	40	40				
		631	1.73	41	41				
	D						1.78	1.78	---
		632	1.12	10	10				
		633	1.52	23	23				
		634	1.69	16	16				
		635	2.20	36	36				
		636	2.19	36	36				
	E						1.81	1.82	0.01
		637	1.17	9	9				
		638	1.59	25	25				
		639	1.78	23	23				
		640	1.94	26	26				
		641	2.11	31	31				
		642	2.50	41	41				

Table A-III: Comparison of Various Color-Difference Formulae on Pooled Phase I and Phase II

				color center			T50 coordinates			color difference metrics							
Color Center			T50	L*	a*	b*	L*	a*	b*	XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
Moderate blue	A	I	0.96	36.11	-1.35	-27.3	37.06	-1.18	-27.3	1.168	0.219	1.239	0.99	0.96	1.207	1.056	0.188
	B	I	1.37	35.85	-1.4	-27.1	35.83	-0.03	-27.2	0.148	0.294	1.704	1.305	1.37	1.54	1.156	1.448
	C	I	1.55	35.86	-1.51	-26.9	35.65	-1.05	-25.5	0.999	0.427	2.627	2.543	1.55	1.878	0.875	0.961
	D	II	1.01	35.92	-1.62	-28.4	35.92	-0.92	-27.6	0.422	0.279	1.84	1.498	1.01	1.22	0.671	0.61
	E	II	3.22	36.29	-1.66	-28.4	36.2	-2.81	-25.4	1.775	0.784	4.425	4.585	3.22	3.349	1.873	1.889
	F	II	1.06	34.72	-1.35	-28.2	35.04	-0.51	-27.7	0.287	0.281	2.037	1.311	1.06	1.235	0.826	0.715
	G	II	1.18	36.22	-1.74	-28.3	37.03	-2.27	-27.6	0.683	0.254	1.769	1.173	1.18	1.072	1.061	0.535
	H	II	0.87	34.77	-1.24	-28	34.25	-0.69	-27.6	0.78	0.217	0.956	1.241	0.87	1.073	0.771	0.468
	I	II	1.53	33.18	-1.09	-28.1	33.84	-0.09	-29	1.219	0.34	1.295	1.814	1.53	1.821	1.243	1.485
Moderate greenish blue	A	I	0.78	50.65	-16.2	-11.2	51.43	-16.2	-11.3	1.264	0.181	0.843	0.791	0.78	1.87	0.711	0.11
	B	I	1.62	50.77	-16.2	-11.1	50.65	-14.6	-11.2	0.228	0.326	2.116	1.812	1.62	3.085	1.136	1.926
	C	I	1.62	50.65	-16.1	-11	50.52	-15.5	-9.49	1.109	0.379	2.688	2.635	1.62	2.282	1.126	1.934
	D	II	1.53	50.77	-16.3	-11.6	50.83	-15	-10.7	0.582	0.34	2.604	2.207	1.53	2.841	0.929	1.335
	E	I	1.48	50.82	-16.2	-11.1	50.85	-16.9	-9.81	0.811	0.32	1.877	2.035	1.48	2.02	1.21	2.044
	F	II	1.24	49.96	-16.7	-11.4	50.54	-15.8	-10.8	0.793	0.272	2.178	1.556	1.24	2.809	0.849	0.963
	G	II	1	49.89	-16.6	-11.4	50.55	-17	-10.7	0.766	0.221	1.253	1.19	1	1.894	0.859	1.012
	H	II	1.19	49.83	-16.4	-11.2	49.11	-15.6	-10.7	1.298	0.27	1.402	1.609	1.19	2.024	0.878	0.835
	I	II	0.87	48.99	-16.8	-11.4	49.69	-16.4	-11.8	1.267	0.195	0.958	0.957	0.87	1.898	0.77	0.707
Medium gray	A	I	0.94	59.78	-0.99	1.25	60.72	-0.95	1.274	1.822	0.221	0.979	0.942	0.94	2.518	0.79	0.16
	B	II	0.87	59.97	-0.93	1.28	59.89	-0.06	1.242	0.169	0.182	1.21	1.162	0.87	2.109	1.193	1.529
	C	I	1.32	59.64	-1.01	1.21	59.6	-0.88	2.523	0.931	0.203	1.948	2.048	1.32	1.921	1.801	2.229
	D	I	0.93	59.55	-1.12	1.07	59.54	-0.41	1.672	0.447	0.191	1.568	1.496	0.93	2.083	1.283	1.623
	E	I	1.27	59.52	-0.98	1.25	59.51	-1.8	2.222	0.696	0.196	1.395	1.588	1.27	2.171	1.723	2.123
	F	II	1.03	58.99	-0.53	0.81	59.74	-0.1	1.364	1.31	0.223	1.407	1.377	1.03	2.631	1.191	1.257
	G	II	1.01	59.09	-0.45	0.9	59.78	-0.88	1.497	1.049	0.196	1.075	1.17	1.01	2.288	1.203	1.302
	H	II	0.87	58.62	-0.49	0.91	58.08	-0.1	1.471	1.217	0.181	1.285	1.239	0.87	1.778	1.079	1.22
	I	II	0.93	58.85	-0.54	0.76	59.47	-0.16	0.188	1.502	0.178	1.011	1.085	0.93	1.882	1.121	1.276
Moderate bluish green	A	I	0.96	55.53	-27.1	2.83	56.49	-27	2.835	1.574	0.224	1.138	0.959	0.96	3.426	0.834	0.143
	A	II	0.98	55.63	-27.4	2.06	56.61	-27.3	2.075	1.616	0.229	1.156	0.982	0.98	3.455	0.851	0.143
	B	I	2.28	55.57	-27.1	2.8	55.58	-24.8	2.595	0.441	0.424	3.113	2.581	2.28	5.795	1.188	1.683
	B	II	2.49	55.47	-27.5	2.1	55.53	-25	1.943	0.521	0.465	3.444	2.813	2.49	6.327	1.291	1.831
	C	I	1.3	55.88	-27.2	2.76	55.87	-26.9	4.013	0.758	0.214	1.996	1.949	1.3	1.95	0.899	1.487
	C	II	1.22	55.9	-27.4	1.98	55.88	-27.1	3.157	0.727	0.202	1.877	1.84	1.22	1.809	0.838	1.411

Table A-III (Cont.):

Color Center			T50	color center L*	a*	b*	T50 coordinates L*	a*	b*	color difference metrics XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
	D	I	1.68	55.63	-27.3	2.76	55.65	-25.8	3.652	0.576	0.331	2.702	2.278	1.68	3.947	0.999	1.56
	D	II	1.78	55.64	-27.6	1.97	55.7	-26.1	2.941	0.612	0.351	2.884	2.42	1.78	4.157	1.049	1.663
	E	I	1.77	55.56	-27.1	2.8	55.65	-28.2	4.117	0.701	0.255	2.055	2.15	1.77	3.316	1.069	1.622
	E	II	1.83	55.61	-27.3	2.15	55.73	-27.5	3.488	0.689	0.196	1.886	1.949	1.349	1.95	0.929	1.532
	F	II	1.28	55.66	-28.3	2.23	56.36	-27.4	2.848	1.049	0.261	2.158	1.556	1.28	4.1	0.874	1.007
	G	II	1.49	55.6	-28.2	2.29	56.52	-28.9	3.23	1.154	0.274	1.543	1.799	1.49	3.749	1.072	1.125
	H	II	1.29	55.44	-28.1	2.22	54.69	-27.2	2.722	1.353	0.278	1.528	1.671	1.29	2.779	0.888	0.933
	I	II	1.17	55.53	-28.2	2.28	56.2	-27.5	1.623	1.444	0.207	1.539	1.267	1.17	3.228	0.811	0.903
Light brown	A	I	0.9	63.54	12.36	20.91	64.43	12.44	20.98	1.785	0.212	0.983	0.942	0.9	2.43	0.732	0.168
	B	I	1.39	63.67	12.37	20.86	63.6	13.75	21.01	0.356	0.295	2.138	2.21	1.39	3.861	1.613	1.651
	C	II	1.62	63.93	12.25	20.64	64.02	12.47	22.24	0.827	0.235	2.029	2.211	1.62	2.558	1.2	1.421
	D	I	1.59	63.61	12.18	20.76	63.58	13.37	21.81	0.679	0.294	2.561	2.604	1.589	3.911	1.081	1.334
	E	I	1.14	63.75	12.17	20.73	63.67	13.02	19.98	0.389	0.212	1.214	1.392	1.141	2.397	1.585	1.576
	F	II	1.49	63.03	12.8	20.09	63.82	13.69	20.99	1.579	0.297	1.849	2.309	1.49	3.587	1.015	1.026
	G	II	1.07	60.5	13.66	21.2	61.27	13.06	21.63	1.236	0.23	1.298	1.22	1.07	2.874	1.242	1.014
	H	II	1.15	61.26	13.2	20.82	60.48	13.83	21.37	1.509	0.238	1.925	1.489	1.15	3.011	0.856	0.705
	I	II	1.01	63.06	12.46	19.13	63.68	13.02	18.56	1.519	0.206	1.078	1.123	1.01	2.081	1.289	1.116
Grayish purple	A	I	0.94	46.48	12.76	-13.4	47 42	12.76	-13.3	1.382	0.216	0.966	0.942	0.94	1.635	0.897	0.146
	B	I	1.47	46.31	12.74	-13.4	46.35	14.2	-13.6	0.342	0.328	1.911	1.809	1.471	2.466	1.125	1.533
	C	II	1.67	46.44	12.61	-13.2	46.5	12.74	-11.5	0.925	0.39	2.758	2.732	1.67	2.246	1.405	2.109
	D	I	1.23	46.52	12.68	-13.4	46.6	13.68	-12.7	0.424	0.286	2.209	1.999	1.23	2.176	1.163	1.662
	E	I	2.88	46.59	12.64	-13.6	46.4	10.89	-11.3	1.609	0.645	3.193	3.64	2.879	3.577	1.829	2.684
	F	II	1.02	46.37	11.52	-12.7	46.87	12.26	-12.3	0.689	0.236	1.664	1.512	1.02	1.787	0.996	1.254
	G	II	1.42	46.5	11.45	-12.7	47.26	10.68	-11.7	0.765	0.314	1.708	1.605	1.42	2.199	1.069	1.134
	H	II	1.03	46.11	11.55	-12.7	45.51	12.28	-12.3	1.003	0.236	1.586	1.442	1.03	1.89	0.999	1.175
	I	II	1.25	46.18	1.43	-12.7	47.05	2.06	-13.4	1.611	0.284	1.164	1.448	1.25	2.099	1.197	1.187
Dark reddish orange	A	I	0.94	42.33	35.94	20.39	43.27	35.91	20.38	1.081	0.215	1.442	0.976	0.94	1.827	0.946	0.092
	B	I	1.94	41.48	35.71	20.41	41.51	37.65	20.41	0.38	0.442	2.956	3.27	1.94	3.345	1.047	1.52
	C	II	1.73	41.56	35.85	21.71	41.52	35.86	23.44	0.433	0.232	1.956	1.819	1.73	2.346	1.245	1.647
	D	I	2.01	41.86	35.78	20.41	41.88	37.16	21.87	0.451	0.352	3.248	3.331	2.01	3.475	0.94	1.359
	E	I	1.32	41.9	35.96	20.46	41.85	35.15	21.5	0.348	0.244	1.227	1.364	1.32	1.703	1.051	1.248
	F	II	1.61	43.11	35.49	22.27	44	36.41	23.25	1.115	0.325	1.382	2.662	1.61	2.111	1.076	0.881
	G	II	0.9	43.99	34.57	23.34	44.42	34.05	23.94	0.404	0.176	1.115	0.966	0.9	1.587	0.787	0.756
	H	II	1.59	43.35	35.28	22.09	42.53	36.21	23.09	0.957	0.286	3.247	2.13	1.59	3.385	1.025	0.901
	I	II	1.25	42.93	35.16	23.56	42.36	34.75	24.59	0.82	0.223	1.415	1.076	1.25	1.699	1.074	1.1
Moderate yellow	A	I	1.19	76.86	4.02	35.14	78.05	4.038	35.17	3.08	0.279	1.45	1.222	1.19	3.665	0.891	0.236

Table A-III (Cont.):

				color center			T50 coordinates			color difference metrics							
Color Center			T50	L*	a*	b*	L*	a*	b*	XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
	B	I	1.44	78.31	1.36	37.23	78.39	2.778	37.46	0.695	0.288	2.137	2.331	1.44	4.581	1.098	1.771
	C	I	2.2	78.47	1.63	35.43	78.76	1.804	37.6	1.319	0.322	2.265	2.729	2.199	3.377	1.022	1.322
	D	I	1.63	78.63	1.16	34.23	78.89	2.198	35.46	1.08	0.273	2.248	2.528	1.63	4.088	1.006	1.487
	E	I	1.28	78.52	1.83	33.65	78.51	2.841	32.87	0.641	0.242	1.381	1.617	1.28	3.129	0.936	1.517
	F	II	1.47	77.87	1.62	36.29	78.65	2.492	37.18	2.115	0.281	1.807	2.2	1.47	4.162	0.968	1.18
	G	II	1.4	77.82	1.99	36.3	78.67	1.372	37.23	1.882	0.274	1.331	1.614	1.4	3.544	0.915	0.996
	H	II	1.3	77.74	2.04	36.14	77.01	2.897	36.79	1.913	0.253	2.042	1.779	1.3	3.645	0.901	1.117
	I	II	1.15	77.99	1.78	36.31	78.64	2.514	35.71	2.078	0.238	1.324	1.319	1.15	3.028	0.816	1.064
Grayish yellow green	A	I	0.86	64.6	-10	13.41	65.45	-10.1	13.44	1.707	0.202	0.926	0.872	0.86	2.782	0.694	0.175
	B	I	1.17	64.73	-9.99	13.49	64.72	-8.83	13.32	0.306	0.232	1.595	1.546	1.17	3.328	0.922	1.436
	C	I	1.44	64.75	-10.3	13.34	64.7	-10.3	14.78	0.974	0.209	1.87	1.997	1.439	2.014	1.075	1.427
	D	I	1.21	64.83	-10.1	13.45	64.63	-9.18	14.25	0.774	0.227	1.928	1.837	1.209	2.895	1.067	1.558
	E	I	1.72	64.74	-9.07	12.06	64.88	-10	13.48	0.791	0.268	1.834	2.124	1.72	3.172	1.196	1.625
	F	II	0.96	65.19	-10.2	13.42	65.82	-9.65	13.91	1.241	0.201	1.341	1.298	0.96	3.049	0.817	0.947
	G	II	1.43	65.21	-10.1	13.36	66.07	-10.9	14.22	1.41	0.274	1.394	1.683	1.43	3.683	1.025	1.048
	H	II	1.09	65.23	-10.2	13.66	64.64	-9.5	14.28	1.389	0.22	1.613	1.508	1.09	2.608	0.935	1.186
	I	II	1.01	65.16	-10.2	13.34	65.8	-9.69	12.74	1.638	0.196	1.201	1.11	1.01	2.613	0.729	0.75
Black	A	II	1.22	14.55	-0.62	0.4	15.77	-0.55	0.375	0.399	0.255	0.735	1.218	1.22	3.102	2.386	0.135
	B	II	0.78	14.07	-0.14	0.48	14.27	0.604	0.467	0.08	0.145	1.168	0.634	0.77	2.159	1.177	1.372
	C	II	1.28	13.89	-0.16	0.74	14.34	-0.56	1.868	0.105	0.179	1.604	1.088	1.28	2.468	1.958	2.108
	F	II	1.1	14	-0.5	0.52	14.91	-0.04	0.926	0.269	0.221	1.252	1.096	1.1	3.064	2.001	1.118
	G	II	1.06	14.42	-0.45	0.44	14.98	-1.08	1.075	0.13	0.174	1.12	0.888	1.06	2.452	1.723	1.608
	H	II	1.05	14.43	-0.34	0.08	13.7	0.246	0.551	0.247	0.207	1.478	1.022	1.05	2.289	1.832	1.399
	I	II	0.94	13.71	-0.96	0.29	14.17	-0.23	-0.07	0.181	0.167	1.101	0.741	0.94	2.376	1.478	1.508
Light bluish green	A	II	1.26	68.77	-30.8	-5.01	70.02	-30.9	-5.04	3.043	0.297	1.461	1.301	1.26	3.808	0.987	0.233
	B	II	2.38	68.59	-30.9	-5.18	68.64	-28.5	-5.28	0.686	0.464	3.111	2.765	2.38	5.464	1.178	1.827
	C	II	1.78	68.4	-30.9	-5.07	68.31	-30.1	-3.5	1.587	0.398	2.781	2.895	1.78	2.846	1.056	1.973
	F	II	1.31	68.42	-30.7	-5.26	69.14	-29.8	-4.62	1.528	0.285	2.199	1.677	1.309	3.747	0.804	0.982
	G	II	1.68	69	-30.8	-4.85	70.03	-31.5	-3.76	1.854	0.372	1.911	2.121	1.68	3.824	1.148	1.36
	H	II	1.67	68.7	-30.6	-4.91	67.88	-29.3	-4.17	2.346	0.361	2.067	2.333	1.67	3.151	0.988	1.245
	I	II	1.47	68.96	-30.6	-4.91	69.76	-30.2	-6.05	2.836	0.33	1.778	2.008	1.47	2.938	1.006	1.484
Moderate reddish brown	A	II	1.01	28.89	21.19	17.64	29.89	21.21	17.75	0.659	0.222	1.208	1.15	1.01	1.966	1.283	0.132
	B	II	1.16	28.84	20.83	17.65	28.87	21.99	17.75	0.135	0.243	1.919	1.706	1.16	2.422	0.891	1.174
	C	II	1.51	28.92	21.1	17.8	28.95	21	19.31	0.206	0.2	1.618	1.334	1.51	1.939	1.42	1.737
	F	II	1.11	28.73	21.12	17.68	29.39	21.7	18.36	0.449	0.214	0.936	1.65	1.11	1.613	0.971	0.688

Table A-III (Cont.):

				color center			T50 coordinates			color difference metrics							
Color Center			T50	L*	a*	b*	L*	a*	b*	XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
	G	I I	0.95	29.17	21.25	17.98	29.72	20.68	18.5	0.299	0.181	1.306	0.95	0.95	1.948	1.106	0.949
	H	I I	1.35	29.01	21.28	17.89	28.12	22.04	18.56	0.56	0.244	2.791	1.384	1.35	3.076	1.247	0.724
	I	I I	0.93	28.69	21.08	17.99	29.27	21.69	17.6	0.448	0.201	0.812	1.111	0.93	1.305	1.08	0.909
Dark bluish green	A	I I	1.07	32.26	-33.2	-5.07	33.27	-32.9	-5.05	0.8	0.225	1.693	1.028	1.07	3.435	1.213	0.242
	B	I I	3.63	31.59	-33.4	-4.95	31.4	-29.8	-5.36	0.209	0.595	4.667	3.013	3.63	6.309	1.76	2.785
	C	I I	1.43	31.6	-33.5	-5.02	31.61	-32.9	-3.74	0.389	0.298	2.458	1.852	1.43	1.892	0.82	1.52
	F	I I	2.24	31.54	-33.6	-5.13	32.29	-31.6	-4.44	0.526	0.388	3.997	1.817	2.24	5.303	1.349	1.502
	G	I I	1.37	31.31	-34	-5.02	32.13	-34.6	-4.07	0.437	0.278	1.81	1.626	1.37	2.99	1.189	1.089
	H	I I	1.51	31.39	-34.1	-4.88	30.61	-32.9	-4.35	0.668	0.31	1.396	1.777	1.51	2.175	1.13	0.96
	I	I I	1.18	31.8	-34.1	-4.88	32.38	-33.3	-5.54	0.627	0.21	1.747	1.124	1.18	2.657	0.896	1.049
Brilliant greenish blue	A	I I	1.23	59.95	-13.7	-25.8	61.17	-13.6	-25.9	2.872	0.289	1.53	1.242	1.23	2.303	1.028	0.24
	B	I I	1.67	59.29	-13.8	-27	59.04	-12.2	-26.7	0.822	0.376	2.187	2.081	1.67	2.474	1.083	1.294
	C	I I	2.07	59.99	-13.1	-25.8	59.64	-12.6	-23.8	2.591	0.534	3.18	3.691	2.07	2.691	1.1	1.428
	F	I I	1	59.51	-13.5	-25.8	60.1	-12.9	-25.4	1.136	0.244	1.769	1.249	1	1.82	0.677	0.59
	G	I I	1.67	60.21	-13.2	-25.6	61.03	-14	-24.3	1.132	0.374	2.25	2.189	1.67	2.255	1.214	1.272
	H	I I	1.36	60.54	-13.2	-25.6	59.63	-12.4	-25	2.506	0.326	1.459	1.932	1.36	2.04	0.947	0.73
	I	I I	2.19	59.84	-13.5	-25.7	60.96	-12.4	-27.2	4	0.494	2.162	2.984	2.19	3.383	1.594	1.733
Very dark red	A	I I	1.06	18.24	25.93	3.26	19.28	26.11	3.177	0.477	0.224	1.302	1.312	1.06	1.576	1.847	0.182
	B	I I	1.73	17.33	25.07	3.29	16.44	26.55	3.362	0.302	0.368	3.58	1.487	1.73	3.308	1.82	1.118
	C	I I	1.63	16.99	25.03	3.83	17.17	25.17	5.445	0.162	0.213	2.301	1.732	1.63	2.554	1.36	1.905
	F	I I	1.42	18.13	26.02	3.59	19	26.86	4.335	0.392	0.283	1.248	2.245	1.42	1.656	1.704	1.035
	G	I I	1.65	18.1	25.77	3.35	19.24	25	4.262	0.415	0.306	2.259	1.493	1.65	2.821	2.218	1.192
	H	I I	1.8	16.21	24.3	3.16	16.24	25.71	4.273	0.15	0.34	3.349	2.358	1.8	3.227	1.139	1.664
	I	I I	1.33	16.5	24.54	3.38	17.1	25.34	2.5	0.332	0.238	1.246	1.317	1.33	1.636	1.445	1.27
Moderate purplish pink	A	I I	1.34	58.42	31.06	-0.16	59.71	31.09	0.189	2.49	0.311	1.478	1.446	1.34	2.578	1.125	0.387
	B	I I	2.41	58.13	31.59	-0.39	58.11	34	-0.28	0.664	0.555	3.504	3.865	2.41	4.654	1.17	1.549
	C	I I	1.91	58.06	31.69	-0.22	57.72	32.25	1.574	1.619	0.333	3.204	3.227	1.91	3.225	1.315	1.747
	F	I I	1.68	58.18	31.35	-0.35	59.08	32.4	0.601	1.75	0.368	2.21	2.858	1.68	2.87	1.137	1.111
	G	I I	1.58	57.58	32	0.28	58.64	31.31	1.229	1.666	0.313	1.847	1.779	1.58	2.963	1.175	1.062
	H	I I	1.58	58.14	31.6	-0.23	57.03	32.48	0.48	2.333	0.354	2.833	2.245	1.58	3.401	1.146	0.87
	I	I I	1.69	58.25	31.27	-0.21	59.46	32.14	-1.01	2.945	0.395	1.584	1.975	1.69	2.491	1.245	0.911
Dark blue	A	I I	1.02	30.13	6.62	-30.8	31.15	6.611	-30.8	1.043	0.226	1.411	1.073	1.02	1.028	1.263	0.078
	B	I I	1.4	30.4	7.23	-31.7	30.31	8.593	-32	0.132	0.3	1.526	1.191	1.4	1.206	1.106	1.598

Table A-III (Cont.):

				color center			T50 coordinates			color difference metrics							
Color Center			T50	L*	a*	b*	L*	a*	b*	XYZ	SVF	NBS	L*u*v*	L*a*b*	FMC2	CMC(1:1)	BFD(1:1)
	C	I I	1.15	30.13	6.61	-30.8	30	6.902	-29.6	0.655	0.335	1.976	1.828	1.15	1.361	0.687	0.563
	F	I I	0.98	30.24	6.87	-31.6	30.63	7.701	-31.2	0.339	0.264	1.803	1.066	0.98	0.918	0.896	0.885
	G	I I	1.73	30.38	6.88	-31.4	31.38	5.96	-30.3	0.635	0.366	2.582	1.529	1.73	1.199	1.488	0.69
	H	I I	1.05	29.93	6.79	-30.8	29.3	7.619	-30.7	0.673	0.232	1.135	1.175	1.05	1.088	1.064	0.922
	I	I I	1.74	30.09	6.78	-31	31.07	7.784	-32	1.537	0.404	1.202	2.176	1.74	2.056	1.499	1.427
Light gray	A	I I	1.3	83.38	0.06	0.13	84.66	-0.14	0.228	4.202	0.301	1.551	1.313	1.3	3.377	0.99	0.507
	B	I I	1.02	84.67	-0.04	0.72	84.64	0.978	0.658	0.385	0.211	1.318	1.448	1.02	2.443	1.497	1.845
	C	I I	1.83	82.69	1.24	0.66	83.24	0.371	2.172	1.383	0.284	1.973	2.497	1.83	3.196	2.475	3.019
	F	I I	1.26	83.89	-0.35	-0.24	84.59	0.442	0.443	2.213	0.27	1.858	1.938	1.26	3.134	1.654	1.947
	G	I I	1.36	83.92	1.02	0	84.63	0.162	0.778	1.757	0.251	1.458	1.674	1.36	2.907	1.739	2.108
	H	I I	1.01	83.43	0.96	0.16	82.93	1.774	0.492	1.784	0.222	1.469	1.51	1.01	2.355	1.308	1.547
	I	I I	1.55	82.39	-0.74	0.07	83.73	-0.62	-0.7	5.092	0.355	1.868	1.816	1.55	3.472	1.498	1.442
Strong orange yellow	A	I I	1.61	75.9	18	79.7	77.46	18.39	79.63	3.894	0.383	1.917	2.099	1.61	4.375	1.199	0.428
	B	I I	1.88	76.11	18.21	79.89	75.98	20.08	79.98	0.565	0.384	2.911	3.336	1.88	5.771	1.133	1.357
	C	I I	4.46	76.19	18.09	79.85	76.24	17.89	84.31	1.057	0.676	2	2.307	4.46	3.136	1.54	1.755
	F	I I	2.1	76.17	18.26	80.01	77.07	19.86	81.03	2.646	0.394	2.414	3.529	2.1	5.168	1.147	1.107
	G	I I	2.06	76.1	18.35	79.81	77.29	17.45	81.23	2.596	0.426	2.145	2.248	2.06	4.672	1.22	1.008
	H	I I	1.78	75.84	18.44	80.11	74.79	19.39	81.19	2.3	0.305	2.594	1.957	1.78	4.664	0.983	0.704
	I	I I	2.1	76.09	18.23	79.89	76.87	19.76	78.68	2.425	0.462	2.136	2.797	2.101	4.81	1.271	1.38
Statistics																	
Num. Cases			156							156	156	156	156	156	156	156	156
Minimum			0.78							0.08	0.145	0.735	0.634	0.77	0.918	0.671	0.078
Maximum			4.46							5.092	0.784	4.667	4.585	4.46	6.327	2.475	3.019
Mean			1.437							1.137	0.291	1.875	1.793	1.433	2.776	1.165	1.19
Stdev			0.513							0.884	0.101	0.729	0.709	0.513	1.076	0.334	0.567

Appendix B: Referenced Color Difference Formulae

NBS unit of color difference (1942)[17]:

$\Delta E_{NBS} = f_g\{[221\bar{Y}^{.25}[(\Delta\alpha)^2 + (\Delta\beta)^2]^{.5}]^2 + [k(\Delta Y^{.5})]^2\}^{.5}$
where

$\bar{Y} = (Y_1 + Y_2)/2$
$\Delta Y^{.5} = Y^{.5}{}_1 - Y^{.5}{}_2$
$\alpha = (2.4266x - 1.3631y - 0.3214)/(1.000x + 2.2633y +1.1054)$
$\beta = (0.5710x + 1.2447y - 0.5708)/(.000x + 2.2633y + 1.1054)$
f_g = gloss factor
k = factor for proximity of samples being compared

Herein value used for $f_g = \bar{Y}/(\bar{Y} + .25)$ and value used for $k = 10$. These were chosen because Hunter[17] used these himself when comparing between various color difference formulae.

FMC-1 or Friele-MacAdam-Chickering Color Difference Formula(1967) [11]

$\Delta E = [(\Delta C)^2 + (\Delta L)^2]^{.5}$
where

$\Delta C = [(\Delta C_1)^2 + (\Delta C_3)^2]^{.5}$
$\Delta L = 0.279(P\Delta P + Q\Delta Q)/aD$
$\Delta C_1 = S(P\Delta P + Q\Delta Q)/bD^2 - \Delta S/b$
$\Delta C_3 = (Q\Delta P \quad P\Delta Q)/aD$
$P = 0.724X + 0.382Y \quad 0.098Z$
$Q = -0.48X + 1.37Y + 0.1276Z$
$S = 0.686Z$
$D = (P^2 + Q^2)^{.5}$
$a^2 = 17.3 \bullet 10^{-6} D^2/[1+2.73P^2Q^2/(P^4 + Q^4)]$
$b^2 = 3.098 \bullet 10^{-4}(S^2 + 0.2015Y^2)$

ΔC_1 represented the yellow-blue component of chromaticness difference, ΔC_3 was the red-green. The ΔL consisted of the "Friele-type" lightness difference. The Y tristimulus value used in the computation of b^2 should be from the standard.

FMC-2 (1968)[11]

$$(\Delta E)^2 = C_{11}(\Delta P)^2 + 2C_{12}\Delta P\Delta Q + C_{22}(\Delta Q)^2 + 2C_{13}\Delta P\Delta S + 2C_{23}\Delta Q\Delta S + C_{33}(\Delta S)^2$$

where

$C_{11} = (e_1^2 + e_3^2)P^2 + e_4^2Q^2$
$C_{12} = (e_1^2 - e_3^2 - e_4^2)PQ$
$C_{22} = (e_1^2 + e_3^2)Q^2 + e_4^2P^2$
$C_{13} = -e_1e_2P$
$C_{23} = -e_1e_2Q$
$C_{33} = e_2^2$
$e_1 = K_1S/bD^2$
$e_2 = K_1/b$
$e_3 = 0.279K_2/aD$
$e_4 = K_1/aD$
$K_1 = 0.55669 + Y\{0.049434 + Y[-0.82575\bullet10^{-3} + Y(0.79172\bullet10^{-5} - 0.30087\bullet10^{-7}Y)]\}$
$K_2 = 0.17548 + Y\{0.027556 + Y[-0.57262\bullet10^{-3} + Y(0.63893\bullet10^{-5} - 0.26731\bullet10^{-7}Y)]\}$
$P = 0.724X + 0.382Y - 0.098Z$
$Q = -0.48X + 1.37Y + 0.1276Z$
$S = 0.686Z$
$D = (P^2 + Q^2)^{.5}$
$a^2 = 17.3\bullet10^{-6}D^2/[1 + 2.73P^2Q^2/(P^4 + Q^4)]$
$b^2 = 3.098\bullet10^{-4}(S^2 + 0.2015Y^2)$

The FMC-2 color difference equation could also have been written as $\Delta E = [K_1(\Delta C)^2 + K_2(\Delta L)^2]^{.5}$ where ΔC and ΔL were defined for the FMC-1 equation above and K_1 and K_2 were as defined here. The approach taken here specified the C_{ij} coefficients which are convenient for using in a matrix equations.

ANLAB 40 (1950)[7,52]

$$\Delta E_{AN40} = [\Delta L^2 + \Delta A^2 + \Delta B^2]^{.5}$$

where

$L = 9.2\ V_Y$
$A = 40(V_X - V_Y)$
$B = 16(V_Y - V_Z)$

$100(X/X_0) = 1.2219V_X - 0.23111V_X^2 + 0.23951V_X^3 - 0.021009V_X^4 + 0.0008404V_X^5$

$100(Y/Y_0) = 1.2219V_Y - 0.23111V_Y^2 + 0.23951V_Y^3 - 0.021009V_Y^4 + 0.0008404V_Y^5$

$100(Z/Z_0) = 1.2219V_Z - 0.23111V_Z^2 + 0.23951V_Z^3 - 0.021009V_Z^4 + 0.0008404V_Z^5$

X_0, Y_0, Z_0 are the tristimulus values of the nominally white object-color stimulus. The A component describes the red-green dimension of the color, B describes the yellow-blue.

CIELAB (1976)[7]

$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{.5}$

where

if $Y/Y_0 > 0.01$ then,

$L^* = 116(Y/Y_0)^{1/3} - 16$

$a^* = 500[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}]$

$b^* = 200[(Y/Y_0) - (Z/Z_0)^{1/3}]$

X_0, Y_0, Z_0 are the tristimulus values of the nominally white object-color stimulus. The a* component describes the red-green dimension of the color, b* describes the yellow-blue.

CIELUV (1976)[7]

$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{.5}$

where

if $Y/Y_0 > 0.01$ then,

$L^* = 116(Y/Y_0)^{1/3} - 16$

$u^* = 13L^*(u' - u'_0)$

$v^* = 13L^*(v' \quad v'_0)$

$u' = 4X / (X + 15Y + 3Z)$

$v' = 9Y / (X + 15Y + 3Z)$

$u'_0 = 4X_0 / (X_0 + 15Y_0 + 3Z_0)$

$v'_0 = 9Y_0 / (X_0 + 15Y_0 + 3Z_0)$

X_0, Y_0, Z_0 are the tristimulus values of the nominally white object-color stimulus. The u* component describes the red-green dimension of the color, v^* describes the yellow-blue.

CMC(l:c) (1984)[14]

$$\Delta E = \left[(\Delta L^*/lS_L)^2 + (\Delta C^*/cS_C)^2 + (\Delta H^*/S_H)^2\right]^{.5}$$

where

if $L_1^* >= 16$ then,

$S_L = 0.40975L_1^*/(1+0.01765L_1^*)$

otherwise,

$S_L = 0.511$

$S_C = 0.0638C_1^*/(1+0.0131C_1^*) + 0.638$

$S_H = S_C(Tf + 1 \quad f)$

$f = \{(C_1^*)^4/(C_1^*)^4 + 1900)\}^{.5}$

if $h < 164°$ or $h > 345°$ then,

$T = 0.36 + |0.4 \cos(h_1 + 35)|$

otherwise,

$T = 0.56 + |0.2 \cos(h_1 + 168)|$

"L_1^*, C_1^* and h_1 refer to the standard of a pair of samples, these values and ΔL^*, ΔC^* and ΔH^* being calculated from the CIELAB formula."

"S_L, S_C, S_H indicate the lengths of the half-axes of the ellipsoid defining unit ΔE."

"l and c are the relative weightings of lightness and chroma required for a particular application." For perceptibility data, $l = c = 1$. For textile and some other critical acceptability usage, $l = 2$, $c = 1$.

SVF (1986)[9]

$$\Delta E = \left[(\Delta F_1)^2 + (\Delta F_2)^2 + (2.3\Delta V_Y)^2\right]^{.5}$$

where

$F_1 = 700\, p_1 - 54p_2$

$F_2 = 96.5p_2$

if $Y > 0.43$ then,

$V_Y = 40\, v_1(Y)$

otherwise,

$V_Y = 0$
$p_1 = v_1(S_1) - v_1(Y)$
if $S_3 <= Y$ then,
$p_2 = v_1(Y) - v_1(S_3)$
otherise,
$p_2 = v_2(Y) - v_2(S_3)$
$v_1(Y) = (Y - 0.43)^{0.51}/((Y-0.43)^{0.51} + 31.75)$
if $Y <= 0.1k(V_Y)$ then,
$v_2(Y) = [Y/k(V_Y)) - 0.1]^{0.86}/[(Y/k(V_Y)) - 0.1]^{0.86} + 103.2$
otherwise,
$v_2(Y) = 0$
$k(V_Y) = 0.140 + 0.175V_Y$
$S_1 = S_1'/S_1'_w$
$S_2 = S_2'/S_2'_w$
$S_3 = S_3'/S_3'_w$
$S_1' = 0.520X + 0.589Y - 0.102Z$
$S_2' = -0.194X + 0.562Y + 0.034Z$
$S_3' = 0.007X - 0.015Y + 0.907Z$

S_1', S_2' and S_3' refer to the relative light absorptions in the three human cone types. $S_1'_w$, $S_2'_w$ and $S_3'_w$ are the same calculations for cone excitations for white light. S_1, S_2 and S_3 are the "centered to white" relative cone excitations.

F_1 and F_2 are "opponent coordinates." "F_1 corresponds roughly with the reddish hue 10 RP and $-F_1$ with greenish hue 10G of the Munsell System; F_2 corresponds with the yellow hue 5Y, and $-F_2$ with the blueish hue 5 PB."

BFD(l:c) (1987)[12,13]

$$\Delta E(BFD) = \{[\Delta L(BFD)/l]^2 + [\Delta C^*/(cD_C)]^2 + (\Delta H^*/D_H)^2 + R_T(\Delta C^*/D_C)(\Delta H^*/D_H)\}^{.5}$$

where

$D_C = 0.035\bar{C}^*/(1 + 0.00365\bar{C}^*) + 0.521$
$D_H = D_C(GT' + 1 - G)$

$G = \{(\bar{C}^*)^4/[(\bar{C}^*)^4 + 14000]\}^{.5}$

$T' = 0.627 + 0.055\cos(\bar{h} - 254°) - 0.040\cos(2\bar{h} - 136°) + 0.070\cos(3\bar{h} - 32°) + 0.049\cos(4\bar{h} + 114°) - 0.015\cos(5\bar{h} - 103°)$

$R_T = R_H R_C$

$R_H = -0.260\cos(\bar{h} - 308°) - 0.379\cos(2\bar{h} - 160°) - 0.636\cos(3\bar{h} + 254°) + 0.226\cos(4\bar{h} + 140°) \; 0.194\cos(5\bar{h} + 280°)$

$R_C = \{(\bar{C}^*)^6/[(\bar{C}^*)^6 + 7x10^7]\}^{.5}$

$L(BFD) = 54.6\log(Y + 1.5) - 9.6$

"The terms $\bar{C}$ and $\bar{h}$ refer to the mean of the C* and h values for the standard and sample, these values and dC* and dH* being calculated from the CIE L*a*b* formula. Care should be taken in calculating dC* and dH*. If dC* is equal to the C* value of sample B minus that of sample A, then the dH* value is positive is A is clockwise relative to sample B on a plot of a* against b*. As in the CMC formula, different *l* and *c* values can be used for different applications. In the present work c=1 was found to be satisfactory in every case."

R_T is the term which "determines the extent of the rotation of the ellipses." The R_C and R_H components of R_T are explained as follows: "The R_C function ... ensures that no discontinuities are introduced into the formula. For $\bar{C}^* = 0$, R_C and therefore R_T are zero. Hence the ellipse for neutral colours is not rotated at all; as $\bar{C}^*$ increases, R_C increases smoothly up to a maximum of one and hence rotation increases smoothly up to a maximum determined by R_H (and is therefore different for different h values)."

NBS unit of color difference (1942)[17]:

```
/* nbs_colordif
       Hunter, R. S., R. W. Harold, The Measurement of Appearance, 2nd Edition, John
       Wiley and Sons, NY, 1987, pp 171-175, pp 119 - 194
*/

double  nbs_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
       int i;
       double  colordif;
       double  *xyz[2];

       float denom, x[2], y[2], Y[2], alpha[2], beta[2], Y_bar_quarter_root,
              delta_alpha_squared, delta_beta_squared, delta_Y_sqrt, fg, K,
              delta_E_nbs, Y_bar;

       xyz[0] = xyz1;
       xyz[1] = xyz2;

       for (j = 0; j < 2; j++) {

              denom = xyz[j][0] + xyz[j][1] + xyz[j][2];
              x[j] = xyz[j][0]/denom;
              y[j] = xyz[j][1]/denom;
              Y[j] = xyz[j][1];

              alpha[j] = (2.4266 * x[j] - 1.363 * y[j] -
                     0.3214) / (x[j] + 2.2633 * y[j] +
                     1.1054);
              beta[j] = (0.5710 * x[j] + 1.244 * y[j]
                     0.5708) / (0 * x[j] + 2.2633 * y[j] +
                     1.1054);

       }

       Y_bar = (Y[0] + Y[1]) / 2.0;
       Y_bar_quarter_root = pow(Y_bar,.25);
       delta_alpha_squared = pow(alpha[0] - alpha[1],2.0);
       delta_beta_squared = pow(beta[0] - beta[1],2.0);
       delta_Y_sqrt = sqrt(Y[0]) - sqrt(Y[1]);
       fg = Y_bar/ (Y_bar + .25);
       K = 10;

       nbs_colordif =
              fg * pow( pow( 221 * Y_bar_quarter_root *
              pow( delta_alpha_squared + delta_beta_squared, .5),2.0) +
              pow( K * delta_Y_sqrt, 2.0), .5);
}
```

FMC-2 (1968)[11]

```
/*  fmc2_colordif
        K. D. Chickering, FMC Color-Difference Formulas: Clarification Concerning Usage,
        J. Opt. Soc. Am. 61, 118-122 (1971)
*/

double  fmc2_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double  C11,C12,C22,C13,C23,C33;
        double  e1,e2,e3,e4;
        double  K1,K2;
        double  P[2],Q[2],S[2],D,a,b,a_sq,b_sq;
        double  *xyz[2];
        double  colordif;

        xyz[0] = xyz1;
        xyz[1] = xyz2;

        K1 = 0.55669 + xyz1[1] * (0.049434 + xyz1[1] * (-.00082575 +
                xyz1[1] * (0.0000079172 - .000000030087 * xyz1[1])));
        K2 = 0.1754 + xyz1[1] * (0.027556 + xyz1[1] * (-.00057262 +
                xyz1[1] * (.0000063893   .000000026731 * xyz1[1])));
        for (i = 0; i < 2; i++) {
                P[i] = .724* xyz[i][0] + .382 * xyz[i][1] + .098 *
                        xyz[i][2];
                Q[i] = -.48* xyz[i][0] + 1.37 * xyz[i][1] + 0.1276 *
                        xyz[i][2];
                S[i] = .686 * xyz[i][2];
        }

        D = pow(P[0] * P[0] + S[0] * S[0], (double)0.5);

        a_sq = .0000173 * D * D/((1 + 2.73 * P[0] * P[0] * Q[0] * Q[0])/
                (pow(P[0],(double) 4.0) + pow(Q[0],(double)4.0)));
        b_sq = .0003098 * (S[0] * S[0] + .2015 * xyz1[1] * xyz1[1]);
        a = sqrt(a_sq);
        b = sqrt(b_sq);

        e1 = K1*S[0]/b/D/D;
        e2 = K1/b;
        e3 = .279 * K2/a/D;
        e4 = K1/a/D;

        C11 = (e1 * e1 + e3 * e3)*P[0]*P[0] + e4*e4*Q[0]*Q[0];
        C12 = (e1 * e1 + e3 * e3 - e4*e4)*P[0]*Q[0];
        C22 = (e1 * e1 + e3 * e3)*Q[0]*Q[0] + e4*e4*P[0]*P[0];
        C13 = -e1*e2*P[0];
        C23 = -e1*e2*Q[0];
        C33 = e2*e2;
```

```
        colordif = sqrt(C11*pow(P[0] - P[1],(double)2.0) +
                2 * C12*(P[0] - P[1])*
                (Q[0] - Q[1]) + C22*pow(Q[0] - Q[1],(double)2.0) +
                2 * C13*(P[0] - P[1]) *
                (S[0] - S[1]) + 2* C23 * (Q[0] - Q[1])*(S[0] - S[1]) +
                C33*pow(S[0] - S[1],(double)2.0));

        return(colordif);
}
```

CIELAB (1976)[7]

```
/* lab_colordif
        Roberson, A., The CIE 1976 Color-Difference Formulae, Color Res. Appl. 2, 7-11
        (1977)
*/

/* external subroutines required
        xyz_to_lab_d65
*/

double lab_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double colordif;
        double *xyz[2],lab[2][3];

        xyz[0] = xyz1;
        xyz[1] = xyz2;

        xyz_to_lab_d65(xyz[0],lab[0]);
        xyz_to_lab_d65(xyz[1],lab[1]);

/* color difference equation */

        colordif = sqrt( pow(lab[0][0]   lab[1][0],2.0) +
                        pow(lab[0][1]   lab[1][1], 2.0) +
                        pow(lab[0][2] - lab[1][2], 2.0) );

        return(color_dif);
}
```

CIELUV (1976)[7]

```
/* luv_colordif
        Roberson, A., The CIE 1976 Color-Difference Formulae, Color Res. Appl. 2, 7-11
        (1977)
*/

/* external subroutines required
```

```
        xyz_to_luv_d65
*/

double  luv_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double  colordif;
        double *xyz[2], luv[2][3];

        xyz[0] = xyz1;
        xyz[1] = xyz2;

        xyz_to_luv_d65(xyz[0],luv[0]);
        xyz_to_luv_d65(xyz[1],luv[1]);

/* color difference equation */

        colordif = sqrt( pow(luv[0][0] - luv[1][0],2.0) +
                         pow(luv[0][1] - luv[1][1], 2.0) +
                         pow(luv[0][2] - luv[1][2], 2.0) );

        return(color_dif);
}
```

CMC(l:c) (1984)[14]

```
/* cmc_colordif
        F. J. J. Clarke, R. McDonald and B. Rigg, Modification to the JPC79 Colour-
        difference Formula, J. Soc. Dyers and Colourists, 100, 128-132 (1984)
*/

/* external subroutines required:
        xyz_to_lab_d65
        lab_to_lch
*/

double  cmc_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double  colordif;
        double  lab[2][3];
        double  lch[2][3];
        double  l_lightness,c_chroma;
        double SL, SC, SH;
        double  T,f;
        double  delta_L, delta_C, delta_H, delta_a, delta_b;
        double  xyz[2][3];
        double  delta_H_squared;

        for (i = 0; i < 3; i++) {
                xyz[0][i] = xyz1[i]/100.0;
                xyz[1][i] = xyz2[i]/100.0;
```

```
	}

/* for verification, examples used white point with XYZ = .9482,1.0,1.073 */
/*	xyz_to_lab(xyz[0],(double).9482,(double)1.000,(double)1.0738,lab[0]);
/*	xyz_to_lab(xyz[1],(double).9482,(double)1.000,(double)1.0738,lab[1]);
*/
	xyz_to_lab_d65(xyz[0],lab[0]);
	xyz_to_lab_d65(xyz[1],lab[1]);


	lab_to_lch(lab[0],lch[0]);
	lab_to_lch(lab[1],lch[1]);



/* for perceptibility, l_lightness = c_chroma = 1 */
	l_lightness = 1;
	c_chroma = 1;

	if (lch[0][0] >= 16) {
		SL = 0.040975*lch[0][0]/(1.0+0.01765*lch[0][0]);
	}
	else {
		SL = 0.511;
	}

	SC = 0.0638 * lch[0][1]/(1+0.0131*lch[0][1]) + 0.638;

/* we have radians. range is between 164 and 345 degrees */
	if ((lch[0][2]<2.86) && (lch[0][2] > -.26)) {
		T = 0.36 + abs(0.4 * cos(lch[0][2] + .61));
	}
	else {
		T = 0.56 + abs(0.2 * cos(lch[0][2] + 2.9));
	}

	f = sqrt(pow(lch[0][1],(double)4.0)/(pow(lch[0][1],(double)4.0) +
		1900));

	SH = SC * (T * f + 1 - f);

	delta_L = lch[0][0]   lch[1][0];
	delta_C = lch[0][1]   lch[1][1];
	delta_a = lab[0][1] - lab[1][1];
	delta_b = lab[0][2]   lab[1][2];
/* due to round off error, sometimes near zero values are negative. Thus check*/
	delta_H_squared = pow(delta_a,(double)2.0) +
		pow(delta_b,(double)2.0)
		pow(delta_C,(double)2.0);
	if (delta_H_squared < 0) delta_H = 0;
	else delta_H = sqrt(delta_H_squared);

	colordif = sqrt(pow(delta_L/l_lightness/SL,(double)2.0) +
		pow(delta_C/c_chroma/SC,(double)2.0) +
```

```
            pow(delta_H/SH,(double)2.0));

        return(colordif);
}
```

SVF (1986)[9]

```
/*   svf_colordif
          Seim, T., A. Valberg, Towards a Uniform Color Space: A Better Formula to Describe
          the Munsell and OSA Color Scales, Color Res. Appl. 11, 11-24 (1986)
*/

double v1();
double k();
double v2();

double  svf_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double  colordif;
        double  xyz[2][3];
        double  F1[2],F2[2],Vy[2];
        double  p1,p2;
        double  S1,S2,S3,S1p,S2p,S3p;
        double  xyz_w[3], S1pw,S2pw,S3pw;
        double  delta_F1, delta_F2, delta_Vy;

/* SVF example uses Illuminant C as reference white point */
/*      xyz_w[0] = .9807;
/*      xyz_w[1] = 1.000;
/*      xyz_w[2] = 1.182;


/* current needs are for a D65 white point */
        xyz_w[0] = .9502;
        xyz_w[1] = 1.000;
        xyz_w[2] = 1.0881;

        for (i = 0; i < 3; i++) {
                xyz[0][i] = xyz1[i];
                xyz[1][i] = xyz2[i];
        }

        S1pw = .52*xyz_w[0] + .589*xyz_w[1] - .102*xyz_w[2];
        S2pw = -.194*xyz_w[0] + .562*xyz_w[1] + .034*xyz_w[2];
        S3pw = .007*xyz_w[0] - .015*xyz_w[1] + .907*xyz_w[2];


        for (i = 0; i < 2; i++) {
                S1p = .52*xyz[i][0] + .589*xyz[i][1] - .102*xyz[i][2];
                S2p = -.194*xyz[i][0] + .562*xyz[i][1] + .034*xyz[i][2];
```

```
            S3p = .007*xyz[i][0] - .015*xyz[i][1] + .907*xyz[i][2];

            S1 = S1p/S1pw;
            S2 = S2p/S2pw;
            S3 = S3p/S3pw;

            if (xyz[i][1] > .43) {
                    Vy[i] = 40 * v1(xyz[i][1]);
            }
            else {
                    Vy[i] = 0;
            }

            p1 = v1(S1) - v1(xyz[i][1]);

            if (S3 <= xyz[i][1]) {
                    p2 = v1(xyz[i][1]) - v1(S3);
            }
            else {
                    p2 = v2(xyz[i][1],Vy[i]) - v2(S3,Vy[i]);
            }

            F1[i] = 700 * p1   54 * p2;
            F2[i] = 96.5 * p2;
        }

        delta_F1 = F1[0]   F1[1];
        delta_F2 = F2[0] - F2[1];
        delta_Vy = Vy[0]   Vy[1];

        colordif = sqrt(pow(delta_F1,(double)2.0) +
                pow(delta_F2,(double)2.0) +
                pow(2.3*delta_Vy,(double)2.0));

        return(colordif);
}


/* auxilary subroutines */

double v1(val)
double val;
{
        if (val >= .43) {
                return(pow(val-.43,(double).51)/(pow(val-.43,(double).51) +
                        31.75));
        }
        else {
                return (0);
        }
}

double k(val)
double val;
```

```
{
        return(.14 + .175*val);
}

double v2(val1, val2)
double val1, val2;
{
        if (val1 >= .1*k(val2)) {
                return(pow(val1/k(val2)-.1,(double).86)/
                        (pow(val1/k(val2) - .1,(double).86) + 103.2));
        }
        else {
                return(0);
        }
}
```

BFD(l:c) (1987)[12,13]

```
/* bfd_colordif
        M. R. Luo and B. Rigg, BFD (l:c) Colour-difference Formula Part 1 - Development of
        the formula, J. Soc. Dyers and Colourists, 103, 86-94 (1987)
        M. R. Luo and B. Rigg, BFD (l:c) Colour-difference Formula Part 2 - Performance of
        the formula, J. Soc. Dyers and Colourists, 103, 126-132 (1987)
*/

/* external subroutines required:
        xyz_to_lab_d65
        lab_to_lch
*/

double bfd_colordif(xyz1,xyz2)
double xyz1[3], xyz2[3];
{
        int i;
        double colordif;
        double xyz[2][3];
        double lab[2][3], lch[2][3];
        double Dc,Dh,G,T_prime,Rt,Rh,Rc,L_BFD[2],dL_BFD;
        double C_bar, h_bar, dL, dC, dH, dE_sq, da, db;
        double dH_sq;
        double l_lightness, c_chroma;

/* for perceptibility, l_lightness = c_chroma = 1 */
        l_lightness = 1;
        c_chroma = 1;

        for (i = 0; i < 3; i++) {
                xyz[0][i] = xyz1[i]/100.0;
                xyz[1][i] = xyz2[i]/100.0;
        }

        for (i = 0; i < 2; i++) {
```

```
                xyz_to_lab_d65(xyz[i],lab[i]);
                lab_to_lch(lab[i],lch[i]);

                L_BFD[i] = 54.6 * log10(xyz[i][1] + 1.5) - 9.6;
        }

        C_bar = (lch[0][1] + lch[1][1])/2.0;
        h_bar = (lch[0][2] + lch[1][2])/2.0;
        dL =    (lch[0][0] - lch[1][0]);
        dC =    (lch[0][1] - lch[1][1]);
        da =    (lab[0][1] - lab[1][1]);
        db =    (lab[0][2]   lab[1][2]);
        dE_sq =         pow(dL,2.0) + pow(da,2.0) + pow(db, 2.0);
/* fix for tiny negative numbers when lch[1][2] == lch[1][2] */
        dH_sq = dE_sq - pow(dL,2.0) - pow(dC,2.0);
        if (dH_sq < 0) dH_sq = 0;
        dH = sqrt(dH_sq);

        dL_BFD = L_BFD[0] - L_BFD[1];

        Rc = sqrt(pow(C_bar,6.0)/(pow(C_bar,6.0) + 7E7));
        Rh = -.260*cos(h_bar - 5.3756)   .379*cos(2*h_bar - 2.7925) -
                .636*cos(3*h_bar + 4.4331) + .226*cos(4*h_bar + 2.4425) -
                .194 * cos(5*h_bar + 4.8869);
        Rt = Rh * Rc;
        T_prime = .627 + .055*cos(h_bar - 4.4331) - .040*cos(2*h_bar - 2.3736) +
                .070 * cos(3*h_bar   .5585) + .049*cos(4*h_bar + 1.9897) -
                .015*cos(5*h_bar    1.7977);

        G = sqrt(pow(C_bar,4.0)/(pow(C_bar,4.0) + 14000));
        Dc = 0.035*C_bar/(1+0.00365*C_bar) + .521;
        Dh = Dc*(G*T_prime + 1 - G);

        colordif = sqrt(pow(dL_BFD/l_lightness,2.0) +
                        pow(dC/c_chroma/Dc, 2.0) +
                        pow(dH/Dh, 2.0) +
                        Rt * (dC/Dc)*(dH/Dh) );

        return(colordif);
}
```